UNIT 7 STUDY GUIDE

TOPIC #1: DISTANCE, MIDPOINT AND SLOPE

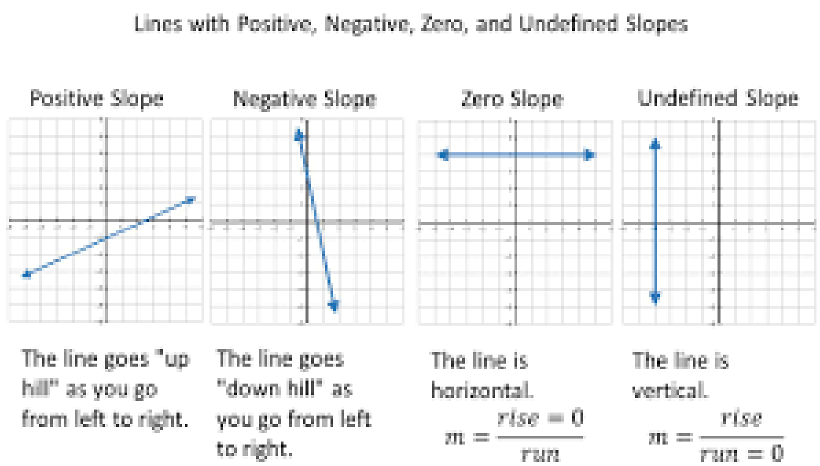
	DISTANCE	SLOPE	MIDPOINT
FORMULA	$d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$	$m = \frac{(y_2 - y_1)}{(x_2 - x_1)}$	$M.P. = \left(\frac{(x_1 + x_2)}{2}, \frac{(y_1 + y_2)}{2}\right)$
KEY WORDS	• **CONGRUENT** • **EQUAL**	• **PARALLEL** *(same slope)* • **PERPENDICULAR** *(negative reciprocal slope)* • **RIGHT ANGLES** *(opposite reciprocal slopes)*	• **BISECT**

MORE ON SLOPE:

Solving for the **slope**: (-4, 3), B (-1,-7)

$$m = \frac{y_2 - y_1}{x_2 - x_1} = \frac{-7-(3)}{-1-(-4)} = \frac{-7-3}{-1+4} = \frac{-10}{3}$$

- **PARALLEL** lines have **EQUAL** slopes.
- **PERPENDICULAR** (*Normal*) lines have **NEGATIVE RECIPROCAL SLOPES**.
- **HORIZONTAL** lines have **ZERO/NO** slope ($y = \#$)
- **VERTICAL** lines have **UNDEFINED** slopes ($x = \#$)



TOPIC #2: DIRECTED LINE SEGMENTS

*In a directed line segment, **ORDER MATTERS**!*

$$(x_1 + k(x_2 - x_1), y_1 + k(y_2 - y_1))$$

BREAKDOWN		EXAMPLE
(x_1, y_1)	The initial (first) point	Find the point on the directed segment from (-4, 5) to (12, 13) that divides it in the ratio of 1:3.
k	$\frac{first\ number\ of\ ratio}{sum\ of\ ratio}$	$(-4+\left(\frac{1}{4}\right)(12--4), (5+\left(\frac{1}{4}\right)(13-5))$ $(-4+4,\ 5+2)$ $(0, 7)$
$x_2, y_2)$	The second (final) point	

TOPIC #3: EQUATION OF A LINE

SLOPE-INTERCEPT FORM	POINT-SLOPE FORM
$y = mx + b$ m = Slope b = Y-Intercept	$y - y_1 = m(x - x_1)$ m = Slope (x_1, y_1) = Point on the line

STEPS FOR WRITING AN EQUATION OF A LINE IN POINT SLOPE FORM WHEN GIVEN THE SLOPE & ONE POINT 1. Substitute the given point (x, y) and (slope) m into $y - y_1 = m(x - x_1)$ 2. Write the equation in terms of $y - y_1 = m(x - x_1)$ 3. Check using the calculator or plug the points	Example: A line having a slope of $-\frac{4}{3}$ and passes through the point (3,-7). Write the equation of this line in **point-slope form**. $m = -\frac{4}{3}$ $x = 3$ $y = -7$ $y - y_1 = m(x - x_1)$ $y - -7 = -\frac{4}{3}(x-3)$ $y + 7 = -\frac{4}{3}(x-3)$
STEPS FOR WRITING AN EQUATION OF A LINE IN POINT SLOPE FORM WHEN GIVEN TWO POINTS 1. Determine the slope. 2. Choose a given point. 3. Substitute the given point (x, y) and (slope) m into $y - y_1 = m(x - x_1)$ 4. Write the equation in terms of $y - y_1 = m(x - x_1)$ 5. Check using the calculator.	Example: Write a linear equation given the two points (1,3) and (8,5) in **point slope form**. 1. $\frac{\Delta y}{\Delta x} = \frac{y_2 - y_1}{x_2 - x_1} = \frac{5-3}{8-1} = \frac{2}{7}$ 2. Point $(1,3)$ 3. $y - y_1 = m(x - x_1)$ $y - 3 = \frac{2}{7}(x-1)$

STEPS FOR WRITING AN EQUATION OF A PERPENDICULAR BISECTOR 1. Determine the slope. 2. Determine the midpoint. 3. Substitute the midpoint (x, y) and the perpendicular slope) m into $y - y_1 = m(x - x_1)$	Example: Write an equation represents the perpendicular bisector of $\overline{AB}$ whose endpoints are $A(8,2)$ and $B(0,6)$. 1. $\frac{\Delta y}{\Delta x} = \frac{y_2 - y_1}{x_2 - x_1} = \frac{6-2}{0-8} = -\frac{4}{8} = -\frac{1}{2} \rightarrow \perp m = 2$ 2. $\left(\frac{x_1+x_2}{2}, \frac{y_1+y_2}{2}\right) \rightarrow \left(\frac{8+0}{2}, \frac{2+2}{2}\right) \rightarrow (4,4)$ 3. $y - y_1 = m(x - x_1)$ $y - 4 = 2(x-4)$ **Answer:** $y - 4 = 2(x-4)$ $y = 2x - 4$

TOPIC #4: COORDINATE PROOFS

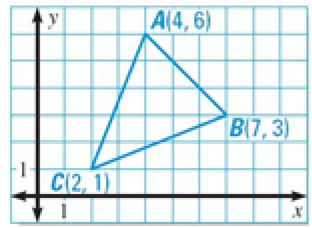
- To prove **<u>ISOSCELES TRIANGLE</u>**-use distance formula **THREE** times to show only **TWO** sides are congruent.
- To prove **<u>EQUILATERAL TRIANGLE</u>**-use distance formula **THREE** times to show all **THREE** sides are congruent.
- To prove **<u>SCALENE TRIANGLE</u>**-use distance formula **THREE** times to show **NO** sides are congruent.

Classify ΔABC as *scalene*, *isosceles*, or *equilateral*.

$$AB = \sqrt{(7-4)^2 + (3-6)^2} = \sqrt{18} = 3\sqrt{2}$$

$$BC = \sqrt{(2-7)^2 + (1-3)^2} = \sqrt{29}$$

$$AC = \sqrt{(2-4)^2 + (1-6)^2} = \sqrt{29}$$



ANSWER

Because $BC = AC$, ΔABC is isosceles.

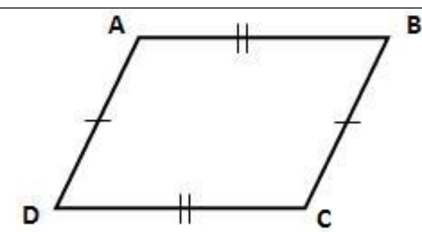
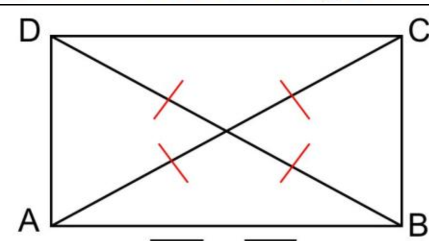
- To prove **<u>RIGHT TRIANGLE</u>**-use distance formula **THREE** times then use **PYTHAGOREAN THEOREM** to show that it is being satisfied.

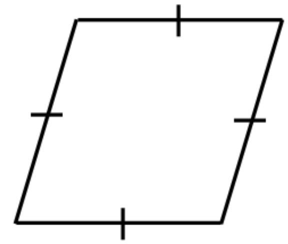
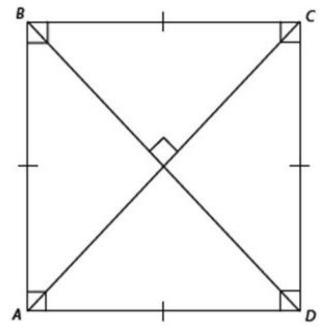
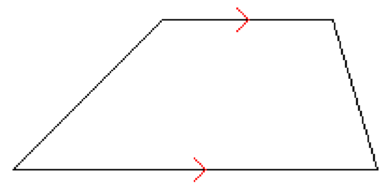
$$a^2 + b^2 = c^2$$

$$(\sqrt{20})^2 + (\sqrt{20})^2 = (\sqrt{40})^2$$

$$20 + 20 = 40$$

$$40 = 40 \checkmark$$

• To prove **<u>PARALLELORAM</u>**- use distance formula **FOUR** times to prove both pairs of **OPPOSITE** sides are congruent.	 *If $\overline{AD} \cong \overline{BC}$ and $\overline{AB} \cong \overline{DC}$,* *then ABCD is a Parallelogram*
• To prove **<u>RECTANGLE</u>**- use distance formula **SIX** times to prove that both pairs of **OPPOSITE** sides are congruent *and* **DIAGONALS** are congruent.	 $\overline{AC} \cong \overline{DB}$

• To prove **<u>RHOMBUS</u>**- use distance formula **FOUR** times to prove that all **FOUR** sides are congruent.	
• To prove **<u>SQUARE</u>**- use distance formula **SIX** times to prove that all **FOUR** sides are congruent *and* **DIAGONALS** are congruent.	
• To prove **<u>TRAPEZOID</u>**- use slope formula **TWO** times to prove that at least one pair of **OPPOSITE** sides are **PARALLEL** *(same slope).*	
• To prove **<u>ISOSCELES TRAPEZOID</u>**- use slope formula **TWO** times to prove that at least one pair of **OPPOSITE** sides are **PARALLEL** *(same slope)*. Then, use distance formula **TWO** times to prove that the non-parallel sides are **CONGRUENT.**