Name: Key   Date: ____________

UNIT 7   **LESSON 9**

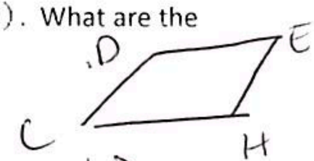
AIM: HOW DO PROVE PARALLELOGRAMS AND RECTANGLES USING COORDINATE GEOMETRY?

Do Now: Fill in the formulas for distance, slope and midpoint on the table below.

	DISTANCE	SLOPE	MIDPOINT
FORMULA	$d=\sqrt{(x_2-x_1)^2+(y_2-y_1)^2}$	$m=\dfrac{y_2-y_1}{x_2-x_1}$	$\left(\dfrac{x_1+x_2}{2}, \dfrac{y_1+y_2}{2}\right)$
KEY WORDS	- congruent - equal	- parallel (same slope) - $\perp$ (opp. recip. slopes)	- Bisect - Diagonals

FORMULA APPLICATIONS:

1. The coordinates of the vertices of parallelogram *CDEH* are $C(\overset{x_1,\ y_1}{-5, 5})$, $D(2,5)$, $E(\overset{x_2\ \ y_2}{-1,-1})$, and $H(-8,-1)$. What are the coordinates of *P*, the point of intersection of diagonals $\overline{CE}$ and $\overline{DH}$?



1) $(-2, 3)$
2) $(-2, 2)$
3) $(-3, 2)$
4) $(-3, -2)$

Diagonals BISEST

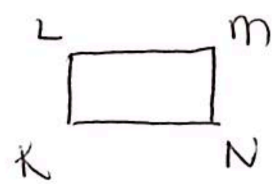
Find midpoint of $\overline{CE}$ or $\overline{DH}$

$\overline{CE}=\left(\dfrac{-5+-1}{2}, \dfrac{5+-1}{2}\right)$ OR $\overline{DH}=\left(\dfrac{2+-8}{2}, \dfrac{5+-1}{2}\right)$

$(-3, 2)$ $(-3, 2)$

2. Rectangle *KLMN* has vertices $K(\overset{x_1,\ y_1}{0, 4})$, $L(4,2)$, $M(\overset{x_2\ \ y_2}{1,-4})$, and $N(-3,-2)$. Determine and state the coordinates of the point of intersection of the diagonals.

Find midpoint of $\overline{KM}$ or $\overline{LN}$

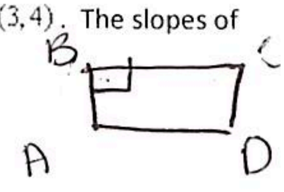


$\overline{KM}=\left(\dfrac{0+1}{2}, \dfrac{4+-4}{2}\right)$ OR $\overline{LN}=\left(\dfrac{4+-3}{2}, \dfrac{2+-2}{2}\right)$

$\left(\frac{1}{2}, 0\right)$ $=\left(\frac{1}{2}, 0\right)$

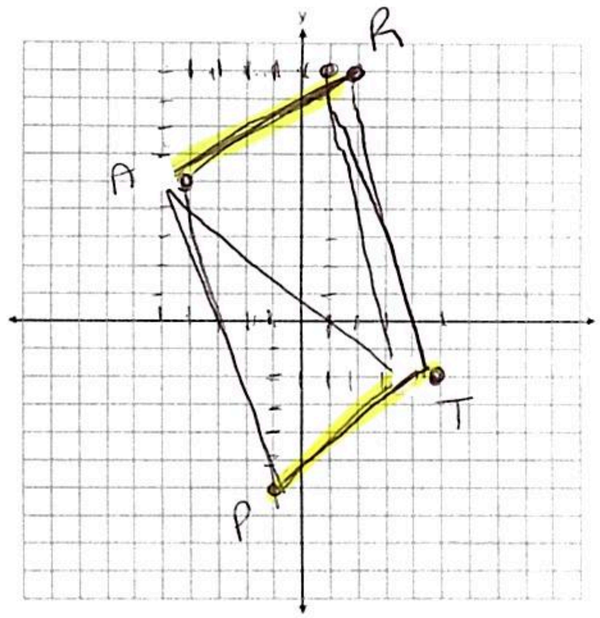
3. The coordinates of the vertices of parallelogram *ABCD* are $A(-3,2)$, $B(-2,-1)$, $C(4,1)$, and $D(3,4)$. The slopes of which line segments could be calculated to show that *ABCD* is a rectangle?

Right $\angle$'s = $\perp$ = slope!



1) $\overline{AB}$ and $\overline{DC}$
2) $\overline{AB}$ and $\overline{BC}$
3) $\overline{AD}$ and $\overline{BC}$
4) $\overline{AC}$ and $\overline{BD}$

4. In the coordinate plane, the vertices of triangle *PAT* are $P(-1,-6)$, $A(-4,5)$, and $T(5,-2)$.



a) State the coordinates of *R* so that quadrilateral *PART* is a parallelogram.

$m_{\overline{PT}} = \frac{4}{6}$ so $m_{\overline{AR}} = \frac{4}{6}$ $\therefore \boxed{(2,9) = R}$

b) Prove that quadrilateral *PART* is a parallelogram.

$\overline{PA} = \sqrt{(-4 - -1)^2 + (5 - -6)^2} = \sqrt{130}$

$\overline{AR} = \sqrt{(2 - -4)^2 + (9-5)^2} = \sqrt{52}$

$\overline{RT} = \sqrt{(5-2)^2 + (-2-9)^2} = \sqrt{130}$

$\overline{PT} = \sqrt{(5 - -1)^2 + (-2 - -6)^2} = \sqrt{52}$

$\overline{PR} = \sqrt{(2 - -1)^2 + (9 - -6)^2} = \sqrt{234}$

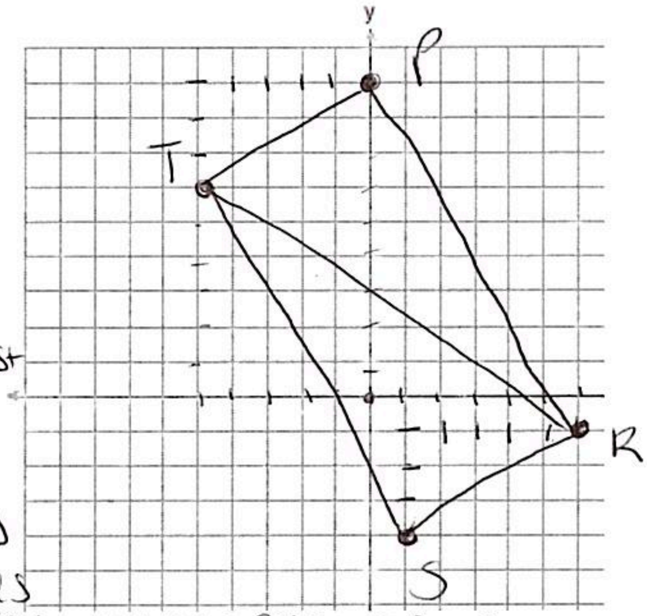
$\overline{AT} = \sqrt{(5 - -4)^2 + (-2-5)^2} = \sqrt{130}$

∴ PART is a $\boxed{P}$ b/c opp. sides are ≅ but diagonals are not

NOTES:

- To find a missing point in a parallelogram or rectangle, use the graph to repeat the ___slope___ on opposite sides.
- To prove a quadrilateral is a parallelogram, we use the ___distance___ formula ___6___ times to show that ___opp. sides are ≅___ but ___diagonals are not___.

6. In the coordinate plane, the vertices of $\triangle RST$ are $R(6,-1)$, $S(1,-4)$, and $T(-5,6)$.



a) Prove that $\triangle RST$ is a right triangle. → 2 ways! slope 3 times or Distance 3 times then $a^2+b^2=c^2$

$\overline{RS} = \sqrt{(1-6)^2+(-4-{-1})^2} = \sqrt{34}$

$\overline{ST} = \sqrt{(-5-1)^2+(6-{-4})^2} = \sqrt{136}$

$\overline{RT} = \sqrt{(-5-6)^2+(6-{-1})^2} = \sqrt{170}$ → 'c' b/c largest side

check: $a^2+b^2=c^2$

$(\sqrt{34})^2+(\sqrt{136})^2=(\sqrt{170})^2$

$34+136=170$

$170=170$ ✓

$\triangle RST$ is a right $\triangle$ b/c the sides satisfy the pythagorean theorem

b) State the coordinates of point *P* such that quadrilateral *RSTP* is a rectangle.

$m_{\overline{SR}} = \frac{3}{5}$ so $m_{TP} = \frac{3}{5}$ so $\boxed{P(0,9)}$

c) Prove that your quadrilateral *RSTP* is a rectangle.

$\overline{RS} = \sqrt{34}$

$\overline{ST} = \sqrt{136}$

$\overline{TP} = \sqrt{(0-{-5})^2+(9-6)^2} = \sqrt{34}$

$\overline{RP} = \sqrt{(0-6)^2+(9-{-1})^2} = \sqrt{136}$

$\overline{RT} = \sqrt{170}$

$\overline{PS} = \sqrt{(0-1)^2+(9-{-4})^2} = \sqrt{170}$

RSTP is a rectangle b/c opp sides and diagonals are $\cong$

- To prove a triangle is a right triangle, we use the distance formula 3 times and use those numbers to prove the Pythagorean theorem $(a^2+b^2=c^2)$. Where 'c' is the largest number.
- To prove a quadrilateral is a rectangle, we use the distance formula 6 times to show that opp sides are $\cong$ and diagonals are $\cong$.

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UNIT 7 **LESSON 9 HOMEWORK**

$x_1 y_1$ $x_2 y_2$

1. Parallelogram *ABCD* has coordinates $A(1,5)$, $B(6,3)$, $C(3,-1)$, and $D(-2,1)$. What are the coordinates of *E*, the intersection of diagonals $\overline{AC}$ and $\overline{BD}$?

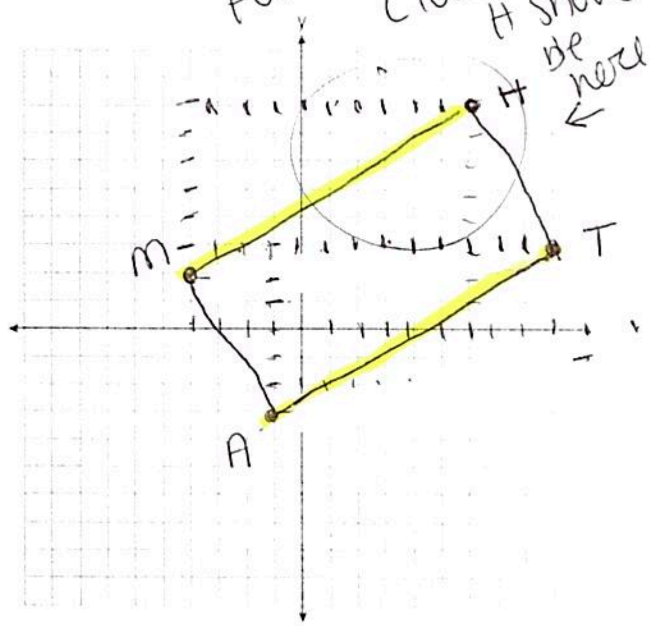
1) $(2,2)$
2) $(4.5,1)$
3) $(3.5,2)$
4) $(-1,3)$

$$\left(\frac{1+3}{2}, \frac{5+-1}{2}\right) = (2,2)$$

2. The vertices of parallelogram *MATH* have coordinates $M(-4,2)$, $A(-1,-3)$, $T(9,3)$. ★BE CAREFUL! Follow letters clockwise H should be here

a) Find the coordinates of point *H* and sketch parallelogram *ABCD* on the accompanying set of axes.

$m_{\overline{AT}} = \frac{6}{10}$ ∴ $m_{\overline{MH}} = \frac{6}{10}$ so $\boxed{H=(6,8)}$



b) Prove that quadrilateral *MATH* is a rectangle.

$\overline{MA} = \sqrt{(-1-{-4})^2 + (-3-2)^2} = \sqrt{34}$

$\overline{AT} = \sqrt{(9-{-1})^2 + (3-{-3})^2} = \sqrt{136}$

$\overline{TH} = \sqrt{(6-9)^2 + (8-3)^2} = \sqrt{34}$

$\overline{MH} = \sqrt{(6-{-4})^2 + (8-2)^2} = \sqrt{136}$

$\overline{MT} = \sqrt{(9-{-4})^2 + (3-2)^2} = \sqrt{170}$

$\overline{AH} = \sqrt{(6-{-1})^2 + (8-{-3})^2} = \sqrt{170}$

∴ MATH is a rectangle b/c opp. sides and diagonals are ≅