Name: Key Date: _____

UNIT 7 **LESSON 2**

AIM: WHAT IS THE RELATIONSHIP BETWEEN SLOPES AND PERPENDICULAR LINES?

Do Now: $x_1 y_1 \quad x_2 y_2$

1. Given points A (-2,4) and B (4,8), which of the following represents the slope of $\overline{AB}$?

A. 2 (B) $\frac{2}{3}$ C. $\frac{-2}{3}$ D. $\frac{3}{2}$ $\frac{8-4}{4-2} = \frac{4}{6} = \frac{2}{3}$

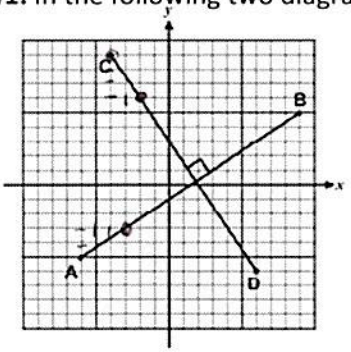
$x_1 y_1 \quad x_2 y_2$

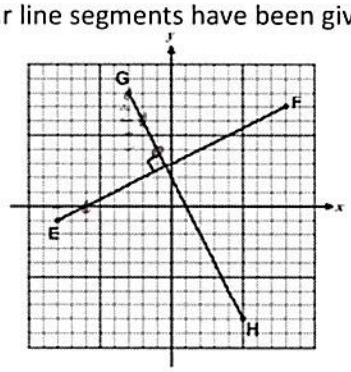
2. A line contains the points (4,-1) and (x,-6) and has a slope of $-\frac{5}{2}$. Find the value of x.

$\frac{-6-1}{x-4} = \frac{-5}{2}$ $-5(x-4) = 2(-5)$ $-5x + 20 = -10$ $-20 \quad -20$ $-5x = -30$ $x = 6$

We saw in the last lesson that two lines with equal slopes are parallel. There is also a connection between lines that are perpendicular and their slopes. We will explore this in the first exercises.

Example #1: In the following two diagrams, two perpendicular line segments have been given.





(a) Determine the slopes of both line segments in both graphs. Label each slope.

Graph #1 **Graph #2**

$m_{\overline{CD}} = \frac{-3}{2}$ $m_{\overline{AB}} = \frac{2}{3}$ $m_{\overline{GH}} = \frac{-2}{1}$ $m_{\overline{EF}} = \frac{1}{2}$

(b) List as many observations as you can about the slopes of these perpendicular lines

opposite reciprocals! - sign change - Fraction Flipped

Discovery	Example
Perpendicular lines have opposite reciprocal slopes!	$m = \frac{1}{2}$ then $\perp m = \frac{-2}{1}$

$x_1 y_1 \quad x_2 y_2$

Example #2: Line segment AB has endpoints of A(–3, 7) and B(3,15) . Which of the following would be the slope of a line perpendicular to AB ?

(A) $\frac{4}{3}$ (B) $-\frac{2}{5}$ (C) $-\frac{3}{4}$ (D) $\frac{3}{7}$

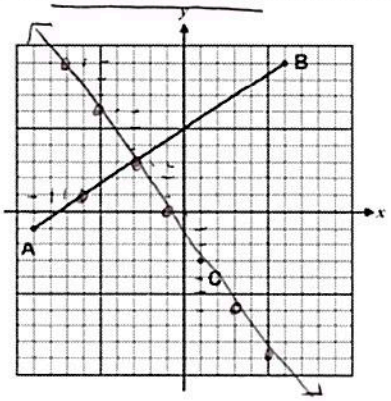
$\frac{15-7}{3-3} = \frac{8}{6} = \frac{4}{3}$

$\perp m = \frac{-3}{4}$

Example #3: If a line was drawn through point C in the diagram such that it is perpendicular to AB, at what coordinate point would the two lines intersect?

$m_{\overline{AB}} = \frac{2}{3}$

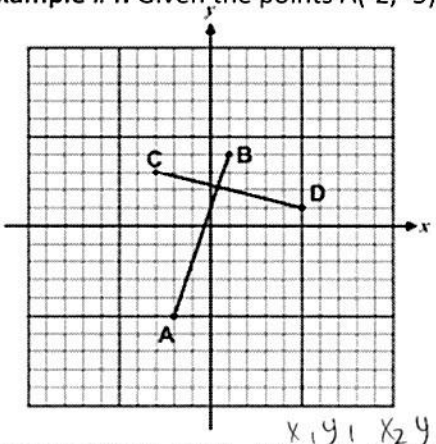
$\perp m = \frac{-3}{2}$



Because of the role of right angles in figures such as right triangles and rectangles, perpendicularity will be extremely important for us. This simple test with slopes will help us determine if right angles are present.

$x_1 y_1 \; x_2 y_2 \quad x_1 y_1 \quad x_2 y_2$

Example #4: Given the points A(-2, -5) B(1, 4) C(-3,3) and D (5, 1) is AB ⊥ CD ? Justify your answer.



$m_{\overline{AB}} = \frac{4-5}{1-2} = \frac{9}{3} = 3$

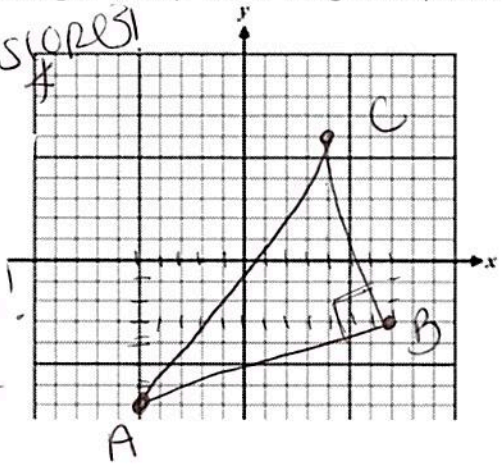
$m_{\overline{CD}} = \frac{1-3}{5-3} = \frac{-2}{8} = -\frac{1}{4}$

NO, the slopes are not opposite reciprocals.

$x_1 y_1 \; x_2 y_2$

Example #5: In ΔABC , A(-5, -7), B(7, -3) and C (4, 6). Is ΔABC a right triangle? Justify. Use of the grid is optional (but advised).

Need one pair of ⊥ sides, check slopes!



$m_{\overline{AB}} = \frac{-3--7}{7-5} = \frac{1}{3}$

$m_{\overline{BC}} = \frac{6--3}{4-7} = -3$

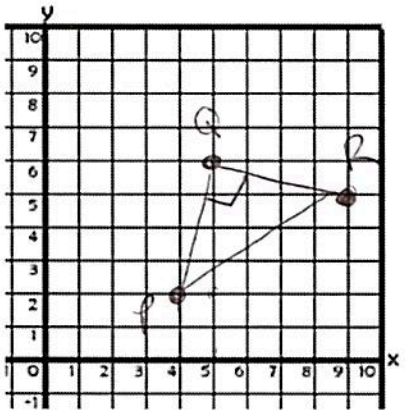
opp. reciprocals!

ΔABC is a right Δ b/c AB ⊥ BC so ∠B is a right ∠.

$m_{\overline{AC}} = \frac{6--7}{4-5} = \frac{13}{9}$

$x_1 y_1$ $x_2 y_2$ $R = (9, 5)$

Example #6: Given triangle PQR with coordinates P(4,2) , Q(5, 6), R(x, 5). Determine the value of x that makes triangle PQR a right triangle with a right angle at Q, then justify your solution.



$\overline{PQ} \perp \overline{QR}$

$m_{\overline{PQ}} = \frac{6-2}{5-4} = \frac{4}{1}$

$\perp m = -\frac{1}{4}$

$m_{\overline{QR}} = \frac{5-6}{x-5} = -\frac{1}{4}$

$-1(x-5) = 4(-1)$

$-x+5 = -4$

$-5 \quad -5$

$-x = -9$

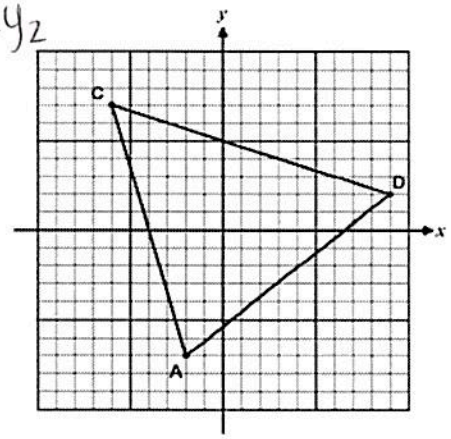
$x = 9$

Altitude

Altitude is drawn perpendicular to the opposite side. ($\perp m$)

Example #7: In the diagram shown, ΔADC has vertices $A(-2,-7)$, $D(9,2)$, and $C(-6,7)$. What is the slope of the altitude drawn from *C* to $\overline{AD}$?

$x_1 y_1$ $x_2 y_2$



need $\perp$ slope of $\overline{AD}$

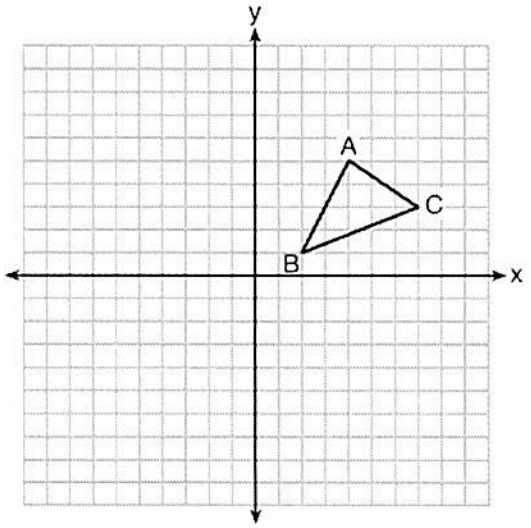
$m_{\overline{AD}} = \frac{2--7}{9--2} = \frac{9}{11}$

$\perp m = -\frac{11}{9}$

Practice NYTS (Now You Try Some!)

1. In the diagram shown, ΔABC has vertices $A(4,5)$, $B(2,1)$, and $C(7,3)$. What is the slope of the altitude drawn from *A* to $\overline{BC}$?

$x_1 y_1$ $x_2 y_2$



need $\perp$ slope of $\overline{BC}$

$m_{\overline{BC}} = \frac{3-1}{7-2} = \frac{2}{5}$

$\perp m = -\frac{5}{2}$

2. If each of the following represents the slope of a line (or line segment), give the slope of a line that is perpendicular to it.

(a) $m = \frac{4}{3}$ $\perp m = \frac{-3}{4}$ (b) $m = -\frac{3}{7}$ $\perp m = \frac{7}{3}$ (c) $m = 1$ $\perp m = -\frac{1}{1} = -1$

3. A line passes through the points E (-1, 4) and F (3, -2). Which of the following is the slope of a line that is perpendicular to $\overleftrightarrow{EF}$?

$x_1 y_1$ $x_2 y_2$

$m = \frac{-2-4}{3-(-1)} = \frac{-6}{4} = \frac{-3}{2}$

$\perp m = \frac{2}{3}$

(1) $\frac{2}{3}$ (2) $\frac{1}{3}$ (3) -3 (4) $-\frac{3}{2}$

4. A line segment whose endpoints are (3, 9) and (7, k) is perpendicular to a line whose slope is -2. Which of the following is the value of k?

$x_1 y_1$ $x_2 y_2$

$\perp m = \frac{1}{2}$

$\frac{k-9}{7-3} = \frac{1}{2}$

$4 = 2(k-9)$

$4 = 2k - 18$

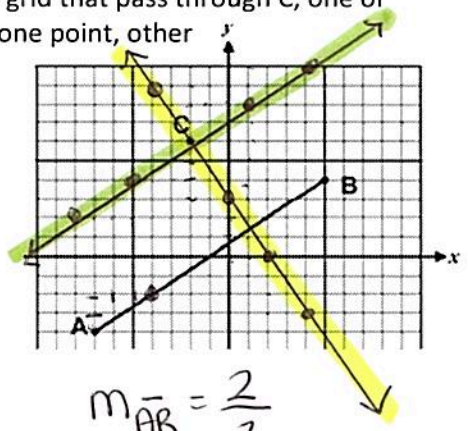
$+18 \quad +18$

$22 = 2k$

$k = 11$

(1) 1 (2) 11 (3) -7 (4) -5

5. On the following grid, AB is shown with along with point C. Draw two lines on this grid that pass through C, one of which is parallel to AB and one that is perpendicular to AB . Give the coordinates of one point, other than C, that lies on each line.



$m_{\overline{AB}} = \frac{2}{3}$

$\perp m = -\frac{3}{2}$

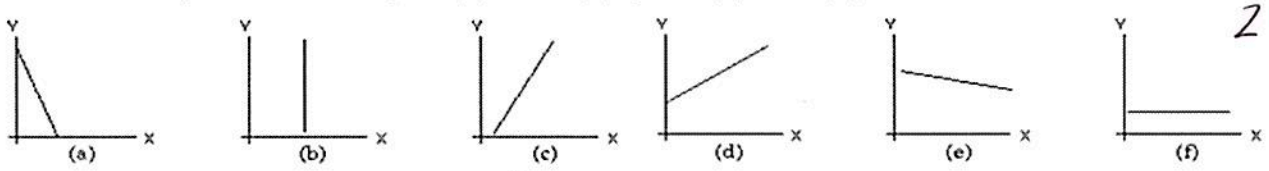
Point that lies on parallel line: (1, 8)

same slope

Point that lies on perpendicular line: (0, 3)

Opp. recip.

6. For a-f, identify whether the slope is (P)ositive, (N)egative, (Z)ero or (U)ndefined.



P U P P N Z

Name: Key

Date:

UNIT 7

LESSON 2 HOMEWORK

If each of the following represents the slope of a line (or line segment), give the slope of a line that is perpendicular to it.

(a) $m = -\frac{1}{3}$ $\perp m = \frac{3}{1}$

(b) $m = \frac{4}{5}$ $\perp m = -\frac{5}{4}$

(c) $m = \frac{4}{1}$ $\perp m = -\frac{1}{4}$

$x_1\ y_1 \quad x_2\ y_2 \quad x_1\ y_1$

2. Are the lines $\overleftrightarrow{AB}$ and $\overleftrightarrow{CD}$ perpendicular if the points defining the line have coordinates of *A* (3, -2), *B* (6, 13), *C* (-5, 8) and *D* (5, 6)? Justify your answer.

$x_2\ y_2$

$m_{\overline{AB}} = \frac{13 - -2}{6 - 3} = \frac{15}{3} = 5$

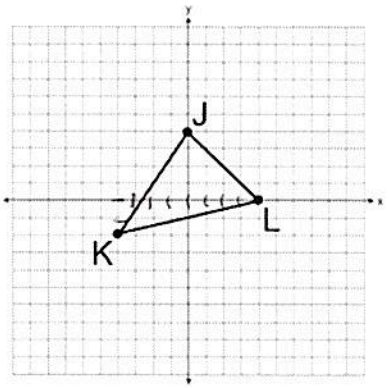
$m_{\overline{CD}} = \frac{6 - 8}{5 - -5} = \frac{-2}{10} = -\frac{1}{5}$

yes! ⊥ lines have opposite reciprocal slopes

3. In the diagram shown, ΔJKL has vertices $J(0,4)$, $K(-4,-2)$, and $L(4,0)$.

What is the slope of the altitude drawn from *J* to $\overline{KL}$?

↳ need ⊥m



$m_{\overline{KL}} = \frac{2}{8} = \frac{1}{4}$

$\perp m = -\frac{4}{1}$

4. In ΔEFG , E(-2, 7), F(7, -8) and G (-6, -3). Is ΔEFG a right triangle? Provide proof of your yes/no answer. The use of the grid is optional.

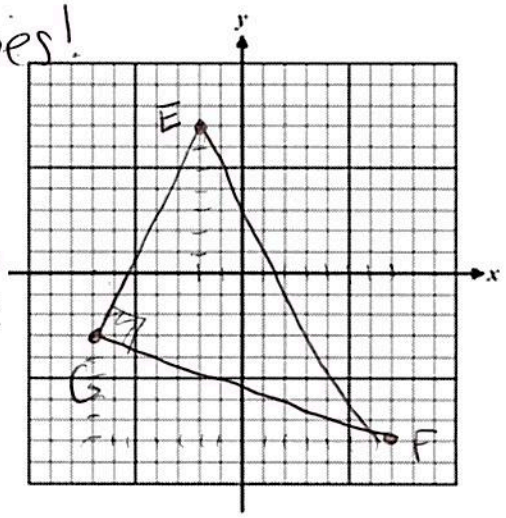
Need one pair of ⊥ sides - check slopes!

$m_{\overline{EG}} = \frac{-3 - 7}{-6 - -2} = \frac{-10}{-4} = \frac{5}{2}$

$m_{\overline{GF}} = \frac{-3 - -8}{-6 - 7} = \frac{5}{-13}$

$m_{\overline{EF}} = \frac{-8 - 7}{7 - -2} = \frac{-15}{9} = -\frac{5}{3}$

NO, ΔEFG is not a right Δ b/c there are no pairs of ⊥ lines



5. The slope of $\overline{QR}$ is $\frac{x-1}{4}$ and the slope of $\overline{ST}$ is $\frac{8}{3}$. If $\overline{QR} \perp \overline{ST}$, determine and state the value of *x*.

$\perp m = -\frac{3}{8}$

$\frac{x-1}{4} = -\frac{3}{8}$

$-12 = 8(x-1)$

$-12 = 8x - 8$

$-4 = 8x$

$x = -2$