Name: Key Date: ______________________

**UNIT 7** **LESSON 10**

## AIM: HOW DO PROVE RHOMBI AND SQUARES USING COORDINATE GEOMETRY?

*Do now:* The diagonals of rhombus *TEAM* intersect at $P(2,1)$. If the equation of the line that contains diagonal $\overline{TA}$ is $y=-x+3$, what is the equation of a line that contains diagonal *EM*? $\perp$ bisector diagonals!

1) $y=x-1$
2) $y=x-3$
3) $y=-x-1$
4) $y=-x-3$

Step 1: slope $=-1$ $\perp m = 1$

Step 2: midpoint $=(2,1)$

Step 3: $y-1=1(x-2)$

$y-1=x-2$

$+1 \qquad +1$

$\boxed{y=x-1}$

**NOTES:**

- A rhombus is a parallelogram with congruent sides, & $\perp$ bisector diagonals.
- A square is a parallelogram with congruent sides, $\cong$ & $\perp$ diagonals and right angles.

$\rightarrow \overline{AC}$ is a diagonal!

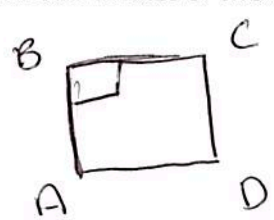
1) Parallelogram *ABCD* has coordinates $A(0,7)$ and $C(2,1)$. Which statement would prove that *ABCD* is a rhombus?

1) ~~The midpoint of $\overline{AC}$ is $(1,4)$.~~ (not enough)
2) ~~The length of $\overline{BD}$ is $\sqrt{40}$.~~ (not $\cong$!)
3) The slope of $\overline{BD}$ is $\frac{1}{3}$.
4) ~~The slope of $\overline{AB}$ is $\frac{1}{3}$.~~ side not diagonal

$m_{\overline{AC}} = \frac{1-7}{2-0} = \frac{-6}{2} = \frac{-3}{1}$ $\perp$ bisector Diagonals

$\perp m = \frac{1}{3}$ ($m\ \overline{BD}$)

2) The coordinates of two vertices of square *ABCD* are $A(2,1)$ and $B(4,4)$. Determine the slope of side $\overline{BC}$.

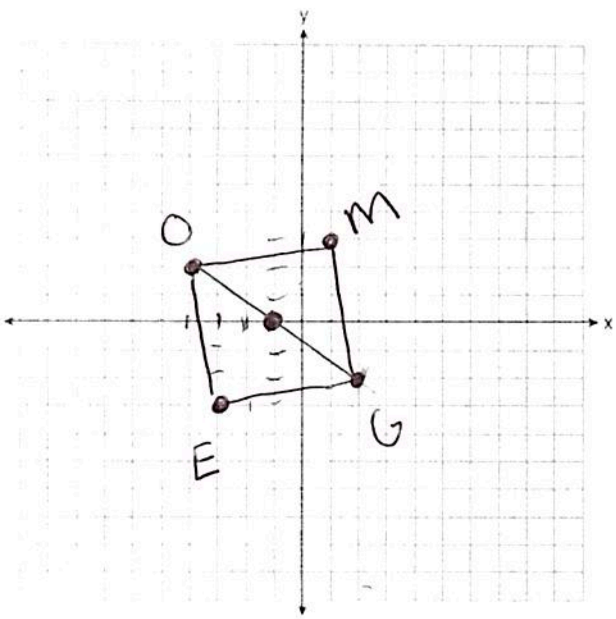



$AB \perp BC \rightarrow$ need opp. recip. slope!

$m_{\overline{AB}} = \frac{4-1}{4-2} = \frac{3}{2}$

$\perp m = \boxed{-\frac{2}{3} = m_{\overline{BC}}}$

3) In square *GEOM*, the coordinates of *G* are $(2,-2)$ and the coordinates of *O* are $(-4,2)$. Determine and state the coordinates of vertices *E* and *M*. [The use of the set of axes below is optional.]




$\overline{GO}$ is a DIAGONAL!

$\perp$ bisector diagonals

Step1: $m\overline{GO} = \frac{2--2}{-4-2} = \frac{4}{-6} = -\frac{2}{3}$

$\perp m = \frac{3}{2}$

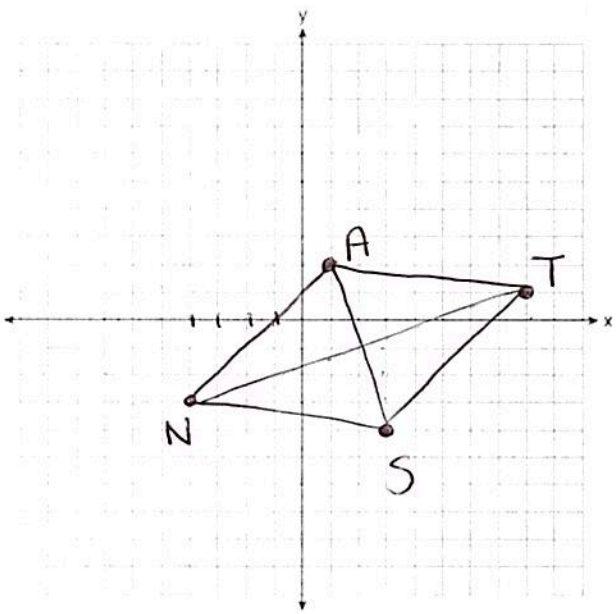
Step2: midpoint: $\left(\frac{2+-4}{2}, \frac{-2+2}{2}\right) = (-1,$

Step3: $y-0=\frac{3}{2}(x+1)$

$y=\frac{3}{2}x+\frac{3}{2}$

$\boxed{\begin{array}{l} G=(-3,-3) \\ M=(1,3) \end{array}}$

4) Quadrilateral *NATS* has coordinates $N(-4,-3)$, $A(1,2)$, $T(8,1)$, and $S(3,-4)$. Prove quadrilateral *NATS* is a rhombus. [The use of the set of axes below is optional.]




$\overline{NA} = \sqrt{(1--4)^2+(2--3)^2} = \sqrt{50}$

$\overline{AT} = \sqrt{(8-1)^2+(1-2)^2} = \sqrt{50}$

$\overline{TS} = \sqrt{(3-8)^2+(-4-1)^2} = \sqrt{50}$

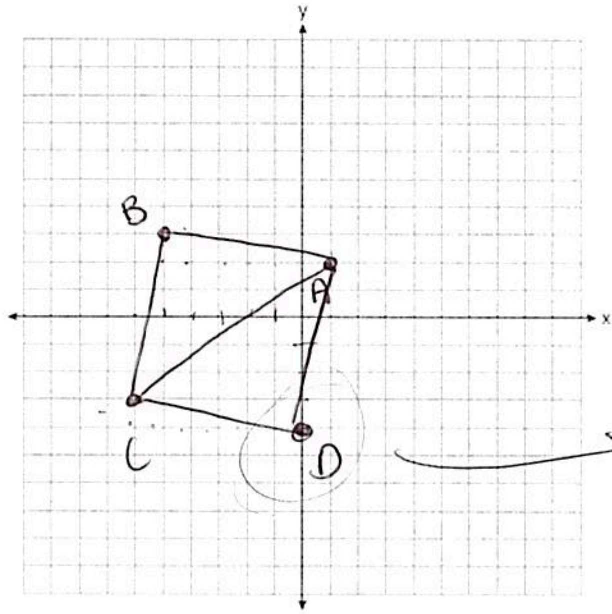
$\overline{NS} = \sqrt{(3--4)^2+(-4--3)^2} = \sqrt{50}$

$\overline{NT} = \sqrt{(8--4)^2+(1--3)^2} = \sqrt{160}$

$\overline{AS} = \sqrt{(3-1)^2+(-4-2)^2} = \sqrt{40}$

**CONCLUSION:** Quad. NATS is a rhombus b/c all sides are $\cong$ but diagonals are not.

5) The coordinates of the vertices of $\triangle ABC$ are $A(1,2)$, $B(-5,3)$, and $C(-6,-3)$. a) Prove that $\triangle ABC$ is isosceles. b) State the coordinates of point $D$ such that quadrilateral $ABCD$ is a square. c) Prove that your quadrilateral $ABCD$ is a square. [The use of the set of axes below is optional.]




$m_{\overline{AB}} = \frac{-1}{6}$

so $m_{\overline{CD}} = -\frac{1}{6}$

$\rightarrow$ D!

a) $\overline{AB} = \sqrt{(-5-1)^2+(3-2)^2} = \sqrt{37}$

$\overline{BC} = \sqrt{(-6-5)^2+(-3-3)^2} = \sqrt{37}$

$\overline{AC} = \sqrt{(-6-1)^2+(-3-2)^2} = \sqrt{74}$

$\triangle ABC$ is isosceles b/c $\overline{AB} \cong \overline{BC}$

b) $D = (0,-4)$

c) $AB = \sqrt{37}$

$\overline{BC} = \sqrt{37}$

$\overline{CD} = \sqrt{(0--6)^2+(-4--3)^2} = \sqrt{37}$

$\overline{AD} = \sqrt{(0-1)^2+(-4-2)^2} = \sqrt{37}$

$\overline{AC} = \sqrt{74}$

$\overline{BD} = \sqrt{(0--5)^2+(-4-3)^2} = \sqrt{74}$

**CONCLUSION:** ABCD is a square b/c all sides and diagonals are $\cong$!

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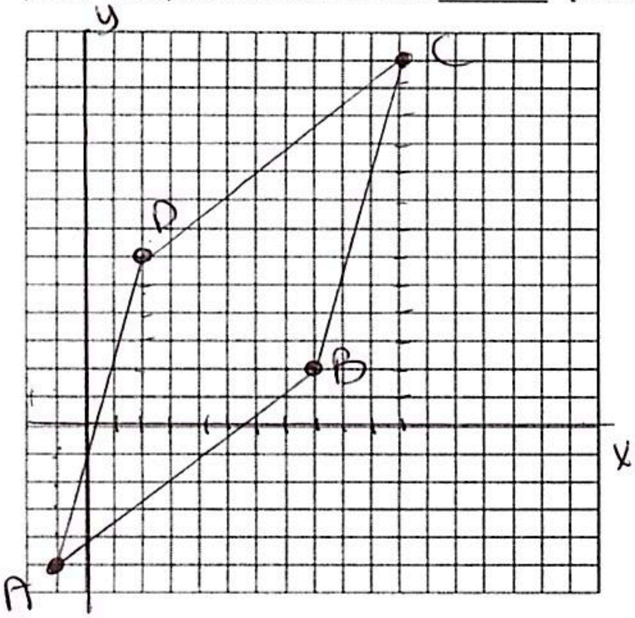
**UNIT 7** **LESSON 10 HOMEWORK**

1) The coordinates of quadrilateral *ABCD* are $A(-1,-5)$, $B(8,2)$, $C(11,13)$, and $D(2,6)$. Using coordinate geometry, prove that quadrilateral *ABCD* is a **rhombus**. [The use of the grid is optional.]




$\overline{AB} = \sqrt{(8--1)^2+(2--5)^2} = \sqrt{130}$

$\overline{BC} = \sqrt{(11-8)^2+(13-2)^2} = \sqrt{130}$

$\overline{CD} = \sqrt{(2-11)^2+(6-13)^2} = \sqrt{130}$

$\overline{AD} = \sqrt{(2--1)^2+(6--5)^2} = \sqrt{130}$

$\overline{AC} = \sqrt{(11--1)^2+(13--5)^2} = \sqrt{468}$

$\overline{BD} = \sqrt{(2-8)^2+(6-2)^2} = \sqrt{52}$

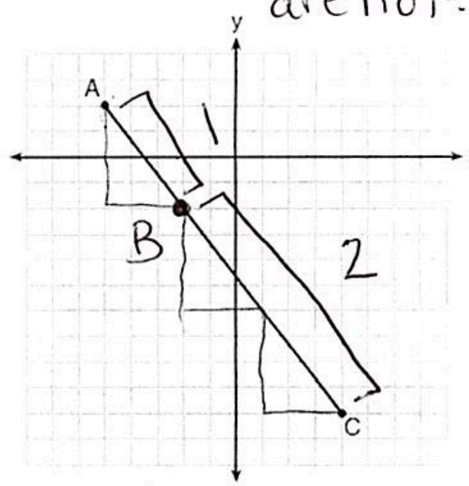
**CONCLUSION:** ABCD is a rhombus b/c all sides are ≅ but diagonals are not.

2) In the diagram below, $\overline{AC}$ has endpoints with coordinates $A(-5,2)$ and $C(4,-10)$. ($x_1, y_1$; $x_2, y_2$)

$k = 1/3$

If *B* is a point on $\overline{AC}$ and $AB:BC = 1:2$, what are the coordinates of *B*?

(1) $(-2,-2)$ 2) $\left(-\frac{1}{2},-4\right)$ 3) $\left(0,-\frac{14}{3}\right)$ 4) $(1,-6)$ DIRECTED line seg!




$(x_1 + k(x_2-x_1),\ y_1 + k(y_2-y_1))$

$(-5+\frac{1}{3}(4--5),\ 2+\frac{1}{3}(-10-2))$

$(-2,-2)$

3) Which equation represents the perpendicular bisector of $\overline{AB}$ whose endpoints are $A(8,2)$ and $B(0,6)$? *(HINT: 3 steps!)*

(1) $y = 2x - 4$

2) $y = -\frac{1}{2}x + 2$

3) $y = -\frac{1}{2}x + 6$

4) $y = 2x - 12$

step 1 = $m = \frac{6-2}{0-8} = \frac{4}{-8} = -\frac{1}{2}$ ⊥m = 2

step 2 = midpoint $\left(\frac{8+0}{2}, \frac{2+6}{2}\right) = (4,4)$

step 3 = $y-4 = 2(x-4)$

$y-4 = 2x-8$

$+4 \quad +4$

$\boxed{y = 2x-4}$