

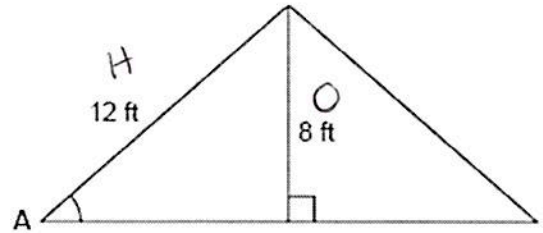
**AIM: APPLICATIONS OF FINDING SIDES AND ANGLES USING SOHCAHTOA**

*Do Now:* The center pole of a tent is 8 feet long, and a side of the tent is 12 feet long as shown in the diagram below. If a right angle is formed where the center pole meets the ground, what is the measure of angle A to the nearest degree?

1) 34  
 ② 42  
 3) 48  
 4) 56

$$\sin^{-1}\left(\frac{8}{12}\right) \sin^{-1}\left(\frac{8}{12}\right)$$

$$A = 41.8103 \approx 42^\circ$$



**Using Trigonometry to Find Missing Angles in a Right Triangle**

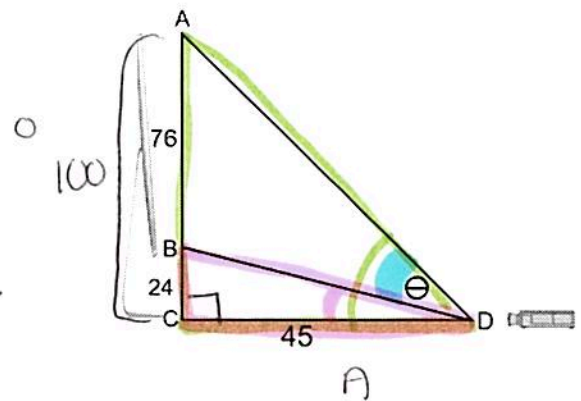
1. Choose sine, cosine, or tangent based on the information.
2. Set up equation.
3. Use inverse trig function (2<sup>nd</sup> sine/cosine/tangent & (Fraction) ).

**Example 1:** As modeled below, a movie is projected onto a large outdoor screen. The bottom of the 76-foot-tall screen is 24 feet off the ground. The projector sits on the ground at a horizontal distance of 45 feet from the screen.

a) Determine and state, to the nearest thousandth of a degree, the measure of  $\angle ADC$ . *\* BIG triangle \**

$$\tan^{-1}(\tan x) = \left(\frac{100}{45}\right) \tan^{-1}$$

$$x = 65.772$$



b) Determine and state, to the nearest thousandth of a degree, the measure of  $\angle BDC$ . *\* small triangle \**

$$\tan^{-1}(\tan y) = \left(\frac{24}{45}\right) \tan^{-1}$$

$$y = 28.072$$

c) Determine and state, to the nearest tenth of a degree, the measure of  $\theta$ , the projection angle.

Plan:  $x - y = \theta$        $65.772 - 28.072 = 37.7^\circ$

**NOW YOU TRY ONE!**

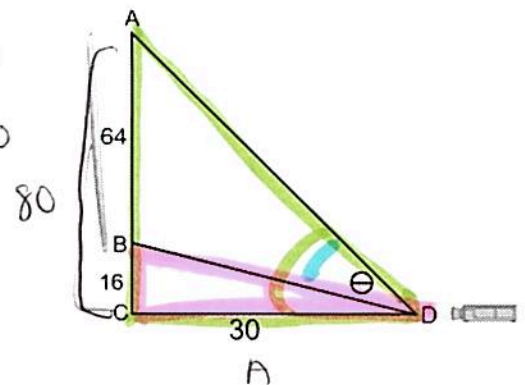
1. As modeled below, a movie is projected onto a large outdoor screen. The bottom of the 64-foot-tall screen is 14 feet off the ground. The projector sits on the ground at a horizontal distance of 30 feet from the screen. Determine and state, to the nearest tenth of a degree, the measure of  $\theta$ , the projection angle. (Hint: Find the big angle of elevation, small angle of elevation and subtract!)

*\* BIG triangle \**      *\* SMALL Δ \**

$$\tan^{-1}(\tan x) = \left(\frac{80}{30}\right) \tan^{-1}$$

$$\tan^{-1}(\tan y) = \left(\frac{16}{30}\right) \tan^{-1}$$

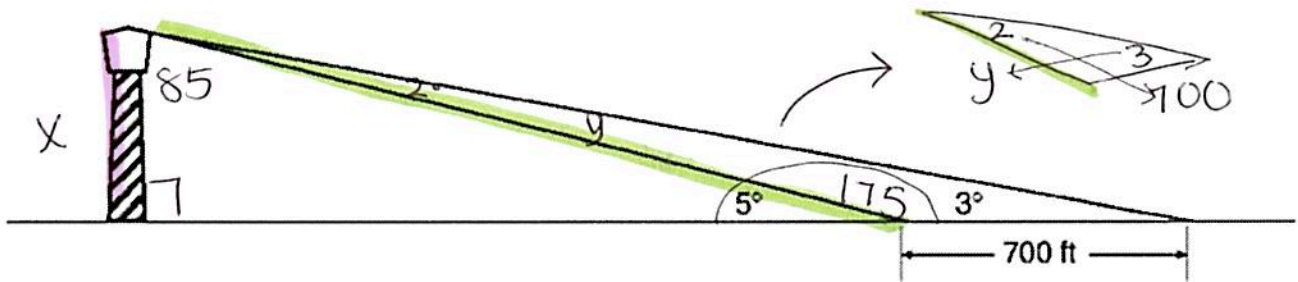
$$x = 69.44395 \quad - \quad y = 28.07248$$



$$\theta \approx 41.4^\circ$$

**WHAT IF WE SEE AN EXAMPLE LIKE THIS?**

**Example 2:** While sailing a boat offshore, Donna see a lighthouse and calculates that the angle of elevation to the top of the lighthouse is  $3^\circ$ , as shown in the accompanying diagram before. When she sails her boat 700 feet closer to the lighthouse, she finds that the angle of elevation is now  $5^\circ$ . How tall, to the nearest tenth of foot, is the lighthouse?

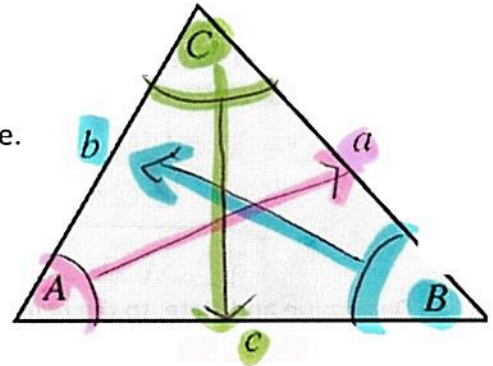


- The **Law of Sines** states that the sine of an angle is proportional to the side opposite the angle.

$$\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$$

- We only use two of the possible ratios (depending on what is given to us), thus producing a proportion we can solve.

- Why use Law of Sines instead of SOHCAHTOA?
  - SOHCAHTOA is restricted to right triangles.
  - Law of Sines can be used in any triangle!



Now, try solving the example above using Law of Sines! *\*FIND ALL ANGLE MEASURES FIRST!\**

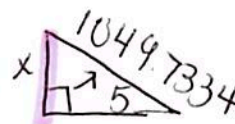
Plan: ① Find side shared between two  $\Delta$ 's (y)

$$\frac{\sin 2}{700} = \frac{\sin 3}{y}$$

$$\frac{y \sin 2}{\sin 2} = \frac{700 \sin 3}{\sin 2}$$

$$y = 1049.7334$$

② use y to help you find x



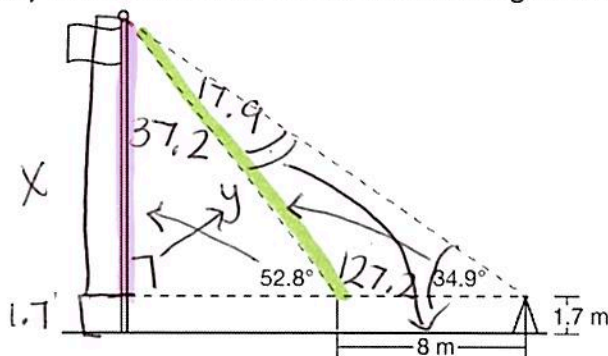
$$\frac{\sin 5}{x} = \frac{\sin 90}{1049.7334}$$

$$\frac{1049.7334 \sin 5}{\sin 90} = \frac{\sin 90 x}{\sin 90}$$

$$x = 91.4902$$

$$\boxed{x = 91.5 \text{ ft}}$$

**Example 3:** Cathy wants to determine the height of the flagpole shown in the diagram below. She uses a survey instrument to measure the angle of elevation to the top of the flagpole, and determines it to be  $34.9^\circ$ . She walks 8 meters closer and determines the new measure of the angle of elevation to be  $52.8^\circ$ . At each measurement, the survey instrument is 1.7 meters above the ground.



Determine and state, to the nearest tenth of a meter, the height of the flagpole. \*add 1.7 to ans.

① Find shared side (y)

$$\frac{\sin 17.9}{8} = \frac{\sin 34.9}{y}$$

$$y \sin 17.9 = \frac{8 \sin 34.9}{\sin 17.9}$$

$$y = 14.8920$$

② Find x

$$\frac{\sin 52.8}{x} = \frac{\sin 90}{14.8920}$$

$$x \sin 90 = \frac{14.8920 \sin 52.8}{\sin 90}$$

$$x = 11.8619$$

$$x = 11.9 + 1.7 = \underline{13.6}$$

### NOW YOU TRY ONE!

The accompanying diagram shows the plans for a cell-phone tower that is to be built near a busy highway. Find the height of the tower, to the nearest foot.

① Find shared side (y)

$$\frac{\sin 32}{y} = \frac{\sin 33}{100}$$

$$y \sin 33 = \frac{100 \sin 32}{\sin 33}$$

$$y = 97.2973$$

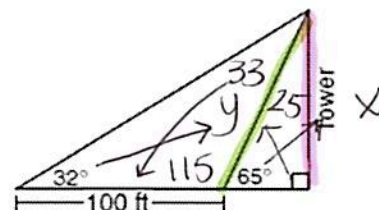
② Find x

$$\frac{\sin 90}{97.2973} = \frac{\sin 65}{x}$$

$$\frac{97.2973 \sin 65}{\sin 90} = \frac{x \sin 90}{\sin 90}$$

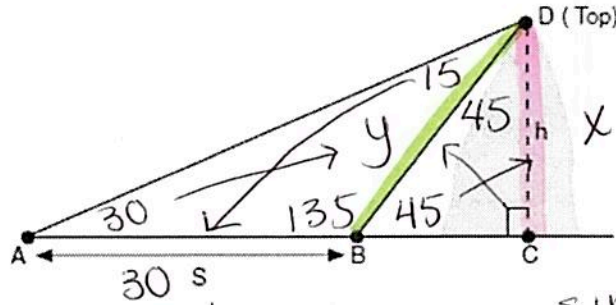
$$x = 88.1813$$

$$\boxed{x = 88 \text{ ft}}$$



**HOMEWORK**

1. A ship at sea heads directly toward a cliff on the shoreline. The accompanying diagram shows the top of the cliff,  $D$ , sighted from two locations,  $A$  and  $B$ , separated by distance  $S$ . If  $m\angle DAC = 30^\circ$ ,  $m\angle DBC = 45^\circ$ , and  $S = 30$  feet, what is the height of the cliff, to the nearest foot?



$$\frac{\sin 30}{y} = \frac{\sin 15}{30}$$

$$y \sin 15 = \frac{30 \sin 30}{\sin 15}$$

$$y = 57.9555$$

$$\frac{\sin 90}{57.9555} = \frac{\sin 45}{x}$$

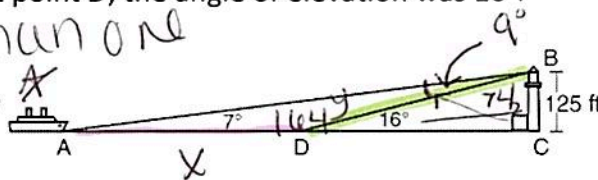
$$x \sin 90 = \frac{57.9555 \sin 45}{\sin 90}$$

$$x = 40.9807$$

$$x \approx 41 \text{ ft}$$

2. As shown in the diagram below, a ship is heading directly toward a lighthouse whose beacon is 125 feet above sea level. At the first sighting, point  $A$ , the angle of elevation from the ship to the light was  $7^\circ$ . A short time later, at point  $D$ , the angle of elevation was  $16^\circ$ .

\*there is more than one way to do this!



\*still start by finding shared side but this time start with  $\triangle BDC$  b/c it has the given

To the nearest foot, determine and state how far the ship traveled from point  $A$  to point  $D$ .

$$\frac{\sin 90}{y} = \frac{\sin 16}{125}$$

$$125 \sin 16 = \frac{y \sin 90}{\sin 16}$$

$$y = 453.4944$$

$$\frac{\sin 9}{x} = \frac{\sin 7}{453.4944}$$

$$x \sin 7 = \frac{453.4944 \sin 9}{\sin 7}$$

$$x = 582.1164$$

$$x \approx 582 \text{ ft}$$

3. As modeled below, a movie is projected onto a large outdoor screen. The bottom of the 40-foot-tall screen is 10 feet off the ground. The projector sits on the ground at a horizontal distance of 25 feet from the screen. Determine and state, to the nearest tenth of a degree, the measure of  $\theta$ , the projection angle.

(Hint: Find the big angle of elevation, small angle of elevation and subtract!)

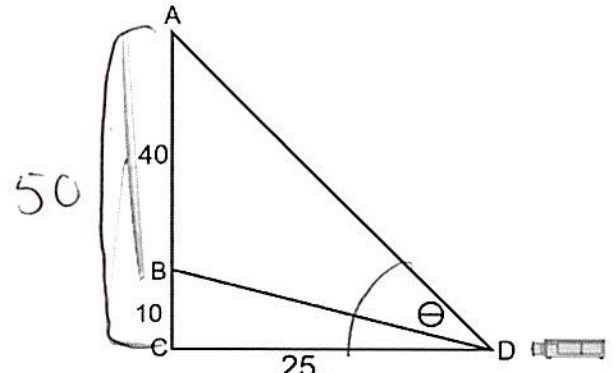
~~$\tan^{-1}(\tan x) = \left(\frac{50}{25}\right) \tan^{-1}$~~

$x = 63.4349$

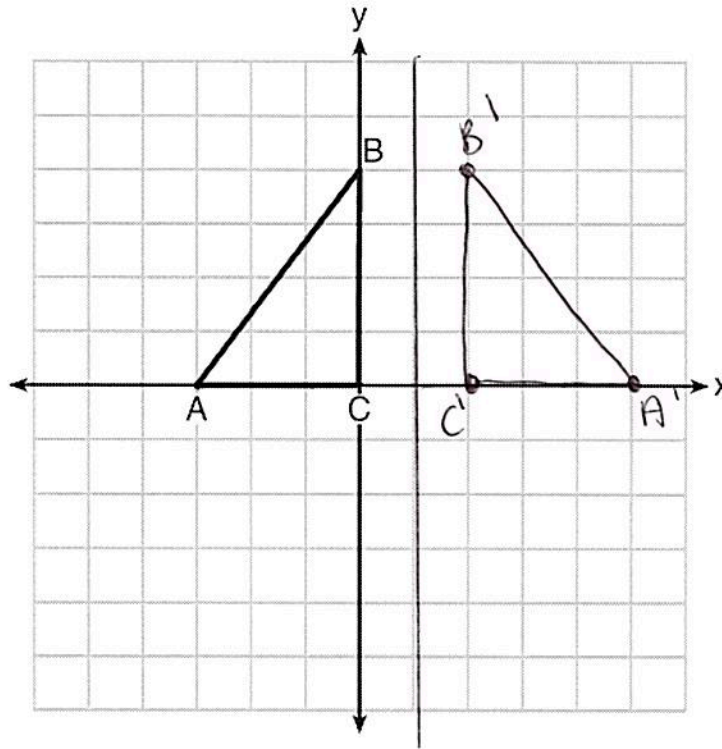
~~$\tan^{-1}(\tan y) = \left(\frac{10}{25}\right) \tan^{-1}$~~

$y = 21.8014$

$\theta = 63.4349 - 21.8014$   
 $\theta = 41.6335 \approx \boxed{41.6^\circ}$



4. Triangle ABC is graphed on the set of axes below. Graph and label  $\Delta A'B'C'$ , the image of  $\Delta ABC$  after a reflection over the line  $x = 1$ .



$\Delta ABC \cong \Delta A'B'C'$  b/c a reflection is a rigid motion which preserves distance + ~~angle~~ measure

