UNIT 6 STUDY SHEET - SIMILARITY

TOPIC #1: SIMILAR TRIANGLES

Similar Triangles

Definition

If corresponding angles are congruent and the ratio of the corresponding sides are in proportion, then the triangles are similar.

Properties/Characteristics

1) All corresponding angles are congruent {AAA similarity}.
2) All corresponding sides are in the same proportion {SSS similarity}.
3) Two pairs of corresponding sides are in the same proportion and the included angle between these corresponding sides are equal {SAS similarity}.

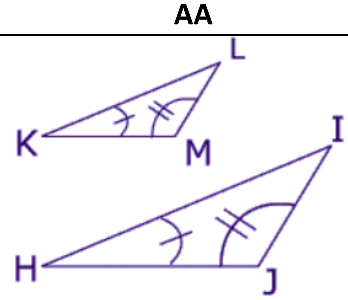
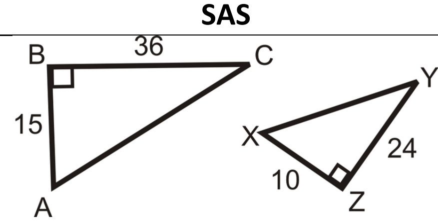
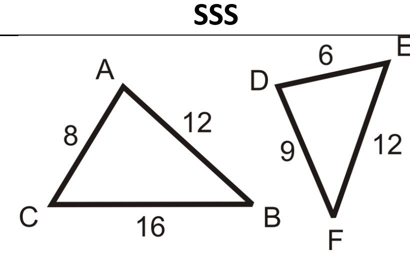
Examples

- A pair of equilateral triangles.
- A pair of isosceles triangles with congruent base angles.
- A pair of isosceles triangles with congruent vertex angles.
- A pair of right triangles with congruent corresponding

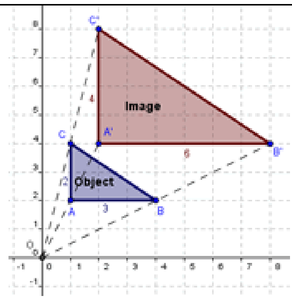
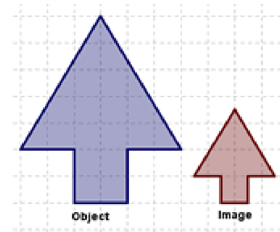
Non-examples

- A square & a rectangle.
- A pair of isosceles triangles with different base angles.
- A pair of isosceles triangles with different vertex angles.

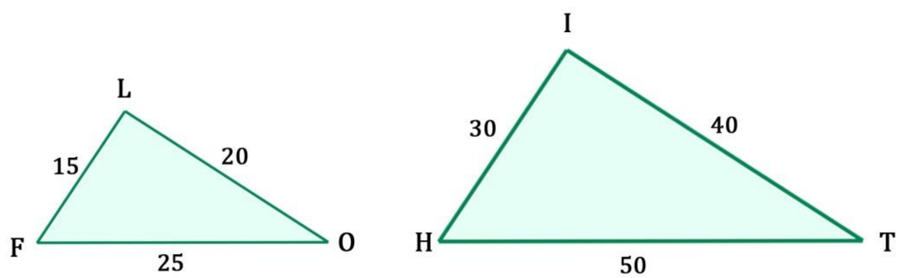
METHODS TO PROVE TRIANGELS ARE SIMILAR:

AA	SAS	SSS
 If two pairs of corresponding angles are congruent, then the triangles are similar $\measuredangle K \cong \measuredangle H$ and $\measuredangle M \cong \measuredangle J$	 If two pairs of corresponding sides are in proportion and one pair of corresponding angles are congruent, then the triangles are similar. $\frac{15}{10} = \frac{36}{24} = \mathbf{\frac{3}{2}}$ and $\measuredangle B \cong \measuredangle Z$	 If all three pairs of corresponding sides share the same ratio (scale factor), then the triangles are similar. $\frac{8}{6} = \frac{12}{9} = \frac{16}{12} = \mathbf{\frac{4}{3}}$

TOPIC #2: SCALE FACTORS $(\boldsymbol{k})$

ENLARGEMENT	REDUCTION
 When you go from small to big, k is greater than 1. $$k = \frac{NEW}{OLD}$$	 When you go from big to small, k is between 0 and 1. $$k = \frac{NEW}{OLD}$$

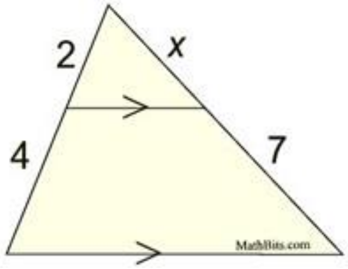
TOPIC #3: RATIO OF THE SIDES, PERIMETERS AND AREAS



RATIO OF THE SIDES	Set up a proportion to find the **SCALE FACTOR** $$\frac{15}{30} = \frac{20}{40} = \frac{25}{50} = \frac{\mathbf{1}}{\mathbf{2}}$$	$1:2$
RATIO OF THE PERIMETERS	This ratio will always be the same as the ratio of the **SIDES** $$\frac{(15 + 20 + 25)}{(30 + 40 + 50)} = \frac{\mathbf{1}}{\mathbf{2}}$$	$1:2$
RATIO OF THE AREAS	This ratio will always be the ratio of the perimeters/sides **SQUARED.** $(1)^2 : (2)^2$	$1:4$

TOPIC #4: SIDE-SPLITTER

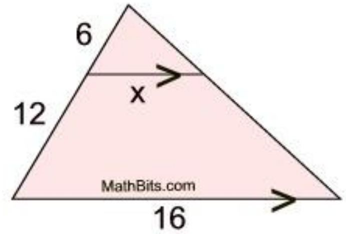
SIDE-SPLITTER



***WHEN THE PARALLEL SIDES ARE **NOT** LABELED, COMPARE THE SIDE LENGTHS)*

$\frac{2}{4} = \frac{x}{7}$ OR $\frac{2}{x} = \frac{4}{7}$ OR $\frac{6}{2} = \frac{x+7}{x}$

SIDE-SPLITTER WITH BASES

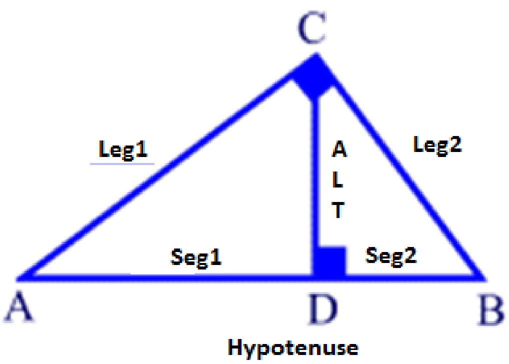


WHEN THE PARALLEL SIDES ARE LABELED, COMPARE THE SMALL TRIANGLE (INSIDE) AND BIG TRIANGLE (OUTSIDE)

$\frac{6}{x} = \frac{18}{16}$ OR $\frac{x}{16} = \frac{6}{18}$

DO NOT USE 12 IN THE PROPORTION!

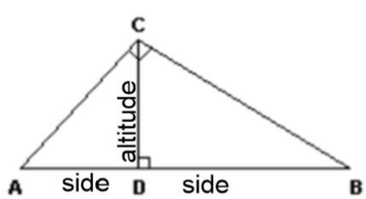
TOPIC #5: RIGHT TRIANGLE PROPORTIONS



*Drawing an **altitude** in a right triangle creates three similar right triangles!*

$\Delta ACB \sim \Delta ADC \sim \Delta CDB$

GEOMETRIC MEAN (ALTITUDE) THEOREM

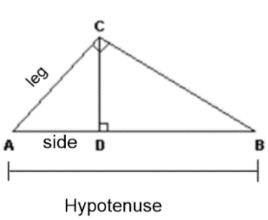


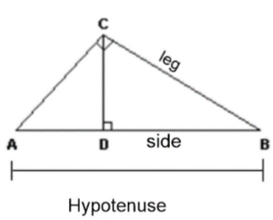
SAAS!

$$\frac{Side_1}{Altitude} = \frac{Altitude}{Side_2}$$

The diagonal of the proportion must be the same number or variable!

GEOMETRIC MEAN (LEG) THEOREM





Use the Hypotenuse of the largest triangle
Use the side closest to the marked leg

HLLS!

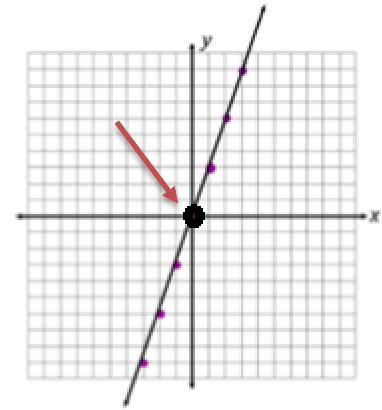
$$\frac{Hypotenuse}{Leg} = \frac{Leg}{Side}$$

The diagonal of the proportion must be the same number or variable!

TOPIC #6: DILATING A LINE

IF THE CENTER OF DILATION IS *ON* THE LINE

The line $y = 3x$ is dilated by a scale factor of 2 and centered at the origin. Write the equation that represents the image of the line after the dilation.

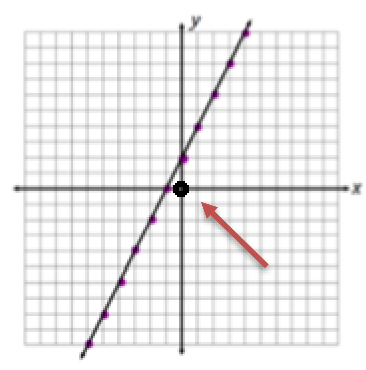


THE RESULTING LINE WILL HAVE THE *SAME* EQUATION!

ANSWER: $y = 3x$

IF THE CENTER OF DILATION IS *OFF* THE LINE

The line $y = 2x + 2$ is dilated by a scale factor of 3 and centered at the origin. Write the equation that represents the image of the line after the dilation.



THE RESULTING LINE WILL HAVE THE *SAME SLOPE* AND *DIFFERENT Y-INTERCEPT*!

To find the y-intercept, multiply the original y-intercept by the scale factor.

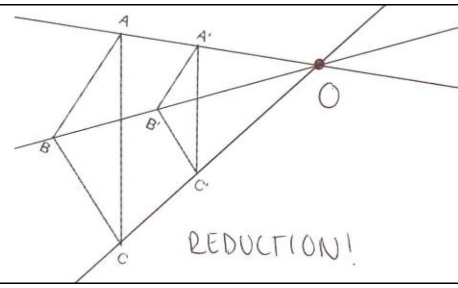
ANSWER: $y = 2x + 2(3)\ or\ y = 2x + 6$

TOPIC #7: SIMILARITY PROOFS

- You can only prove triangles are similar using ***AA***!
- Examples of congruent angles could be: reflexive, right angles, vertical angles, alternate interior angles, corresponding angles, etc.
- After you prove there are two pairs of corresponding congruent angles, complete the following statements/reasons:

	PROVE STATEMENT	REASON
1.	Similarity Statement $\Delta ABC \sim \Delta DEF$	$AA \cong AA$
2.	Proportion $\frac{AB}{BC} = \frac{DE}{EF}$	Corresponding sides of similar triangles are in proportion.
3.	Product $BCxDE = ABxEF$	The product of the means equals the product of the extremes

TOPIC #8: SCALE DRAWINGS

	INSTRUCTIONS	EXAMPLE
CENTER OF DILATION	1. Connect all corresponding points from big triangle to small triangle. 2. Label the point of intersection the center of dilation.	
TRIANGLES	1. Extend the lines stemming from the center of dilation. 2. If r > 1, measure the distance between the C.O.D and each point of the triangle. 3. Move your compass and extend the length along the line according to the scale factor. 4. If 0 < r < 1, construct perpendicular bisectors between each side length. 5. Connect the new points