Name: Key

Date: ____________________

UNIT 5

REVIEW

SIMILARITY REVIEW!

Pre-Image & Image Relationship	Side Relationships	
	Lengths	**Parallel or Perpendicular**
After a dilation($k \neq 1$) the two figures will be similar. (congruent or similar?)	Image = k(pre-image) or Pre-image = $\frac{1}{k}$(image)	Image || Pre-image (|| or $\perp$?)

1. Which transformation below shows a dilation?

1) $(x, y) \rightarrow (y, x)$
2) $(x, y) \rightarrow (\frac{1}{2}x, \frac{1}{2}y)$
3) $(x, y) \rightarrow (x + 2.5, y - 4)$
4) $(x, y) \rightarrow (-x, y)$

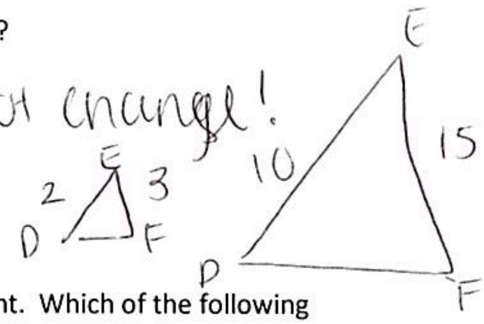
2. If ΔDEF is dilated by a factor of 5, which of the following statements would be true?

~~(1)~~ $m\angle D' = 5 \cdot m\angle D$

~~(3)~~ $m\angle E = \frac{1}{5} \cdot m\angle E'$ → ∠'s do not change!

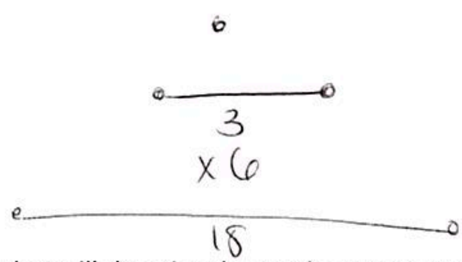
(2) $DE = \frac{1}{5}D'E'$

(4) $EF = 5E'F'$



3. A three-inch line segment is dilated by a scale factor of 6 and centered off the segment. Which of the following statements is true of the image?

1) Length is 18 inches and is parallel to the original
~~2) Length is 18 inches and is perpendicular to the original~~
3) Length is 9 inches and is parallel to the original
~~4) Length is 9 inches and is perpendicular to the original~~



4. The image of ΔDEF is $\Delta D'E'F$. Under which transformation will the triangles *not* be congruent?

~~1)~~ a reflection through the origin

~~2)~~ a reflection over the line $y = x$

rigid motions

3) a dilation with a scale factor of 1 centered at (2, 3) → doesn't change size

4) a dilation with a scale factor of $\frac{3}{2}$ centered at the origin

5. The coordinates of ΔABC are $A(1, 1)$, $B(2, 3)$, and $C(3, 1)$. If $\Delta A'B'C'$ is the image of ΔABC after a dilation centered at the origin with a scale factor of 2 followed by a reflection over the line y = x, then $\Delta A'B'C'$ is

1) an equilateral triangle
2) congruent to ΔABC
3) a right triangle
4) similar to ΔABC

Describing Transformations!

Reflection needs	Rotation needs	Translation needs	Dilation needs
• Line of Reflection	• Center • Angle • Direction	• Distance • Direction	• Center • Scale Factor

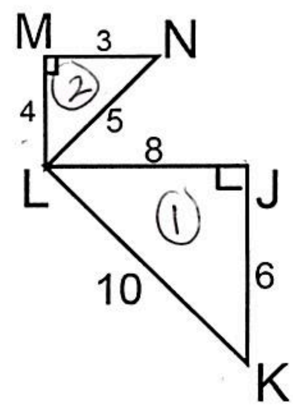
6. Which sequence of transformations will map $\overset{1}{\Delta JKL}$ onto $\overset{2}{\Delta MNL}$?

~~1)~~ Rotation 90° CCW around L, followed by a dilation scale factor of 2 centered at L.

2) Rotation 90° CCW around L, followed by a dilation scale factor of $\frac{1}{2}$ centered at L.

~~3)~~ Reflection over $\overline{LJ}$, followed by a dilation scale factor of 2 centered at L.

4) Rotation 180° CCW around L, followed by a dilation scale factor of $\frac{1}{2}$ centered at L.



Scale Factor (k)

$$k = \frac{\text{New}}{\text{Old}}$$

7. In the diagram below, $\overline{CD}$ is the image of $\overline{AB}$ after a dilation of scale factor *k* with center *O*. Which ratio is equal to the scale factor *k* of the dilation?

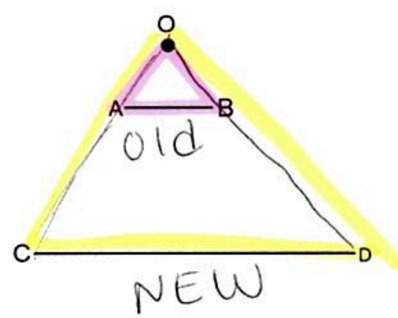
1) $\frac{OC}{OA}$

2) $\frac{BA}{OA}$

3) $\frac{OA}{BA}$

4) $\frac{OA}{OC}$

$\frac{\text{new}}{\text{old}}$



8. In the diagram below, ΔABE is the image of ΔACD after a dilation centered at the origin. The coordinates of the vertices are $A(0,0)$, $B(3,0)$, $C(4.5,0)$, $D(0,6)$, and $E(0,4)$. The scale factor is

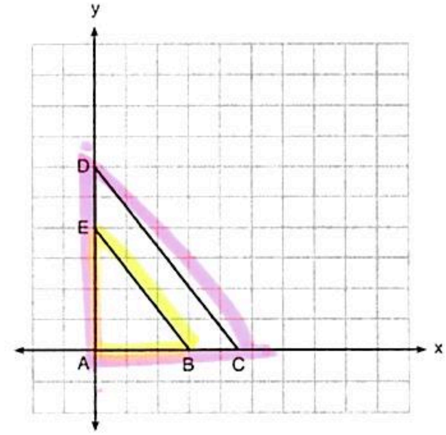
1) $\frac{CD}{BA}$

2) $\frac{AE}{AD}$

3) $\frac{AD}{AE}$

4) $\frac{AD}{DC}$

$\frac{\text{new}}{\text{old}}$



Dilating Lines and Segments

If CENTER ON THE LINE	If CENTER OFF THE LINE
Keep slope **&** y **-** intercept **the SAME.** ☆ same equation! ☆	**Keep the** slope **the same.** (because image is parallel to pre-image) **Multiply the** y **-** intercept **by the scale factor (k).**

9. Line ℓ is mapped onto line m by a dilation centered at the origin with a scale factor of 3. The equation of line ℓ is $2x - y = 8$. Determine and state an equation for line m.

↳ need $y = mx + b$!

$2x - y = 8$
$-2x \quad\quad -2x$
$-y = -2x + 8$

→ $y = 2x(-8 \times 3) = -24$

put in calc!

x	y
0	-8

$\boxed{y = 2x - 24}$

10. Line $y = 3x - 1$ is transformed by a dilation with a scale factor of 2 and centered at $(3, 8)$. The line's image is

1) $y = 3x - 8$ ↳ use calc!
2) $y = 3x - 4$
3) $y = 3x - 2$
4) $y = 3x - 1$

↓ on the line!

Ratio of Areas

Ratio Sides/Perimeters = $x : y$ ex) $(1:4)^2$

Ratio Areas = $x^2 : y^2$ $1:16$

11. The sides of a triangle are 8, 12 and 15. The longest side of a similar triangle is 18. What is the ratio of the ~~perimeter~~ area of the smaller triangle to the ~~perimeter~~ area of the larger triangle?

a. 2:3
b. 4:9
c. 5:6
d. 25:36

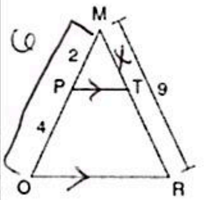
$\frac{15}{18} = \frac{5}{6}$

$5:6$

$5^2 : 6^2$

$25:36$

Side Splitter	Side Splitter with BASES
$\overline{PT} \parallel \overline{OR}$, find the length of MT.	**MUST USE SMALL TRIANGLE AND BIG TRIANGLE!** $\overline{AC} \parallel \overline{DE}$. If $AD = 24$, $DB = 12$, and $DE = 4$, what is the length of AC

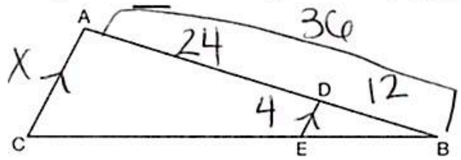


$\frac{6}{2} = \frac{9}{x}$

$6x = 18$

$x = 3$

$\boxed{MT = 3}$



$\frac{36}{x} = \frac{12}{4}$

$12x = 144$

$x = 12$

$\boxed{AC = 12}$

12. In the diagram below, triangle ACD has points B and E on sides $\overline{AC}$ and $\overline{AD}$, respectively, such that $\overline{BE} \parallel \overline{CD}$, $AB = 1$, $BC = 3.5$, and $AD = 18$. What is the length of $\overline{AE}$, to the *nearest tenth*?

1) 14.0

2) 5.1

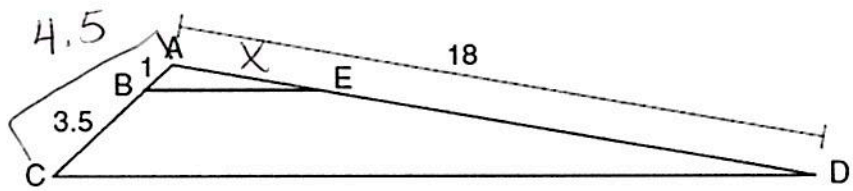
3) 3.3

4) 4.0

$\frac{4.5}{1} = \frac{18}{x}$

$\frac{4.5x}{4.5} = \frac{18}{4.5}$

$x = 4$



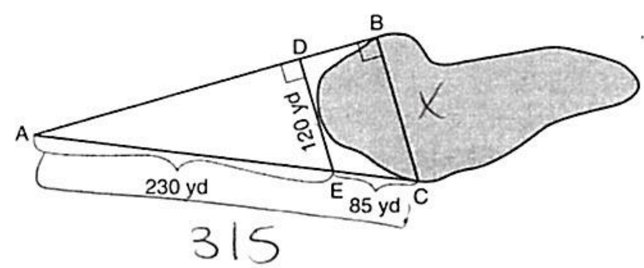
13. To find the distance across a pond from point B to point C, a surveyor drew the diagram below. The measurements he made are indicated on his diagram. Use the surveyor's information to determine and state the distance from point B to point C, to the *nearest yard*.

$\frac{230}{120} = \frac{315}{x}$

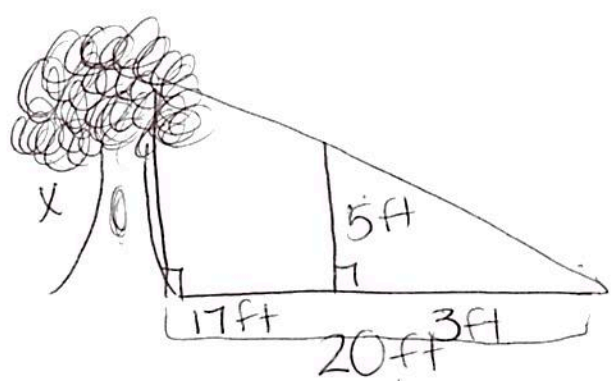
$230x = 37800$

$x = 164.3478$

$\boxed{BC = 164 \text{ yds}}$



14. A tree casts a shadow 20 feet long. A vertical pole stands at a distance of 17 feet from the base of the tree, such that the end of the pole's shadow meets the end of the tree's shadow. If the pole is 5 feet tall, determine and state the height of the tree to the *nearest tenth of a foot*.



$\frac{20}{x} = \frac{3}{5}$

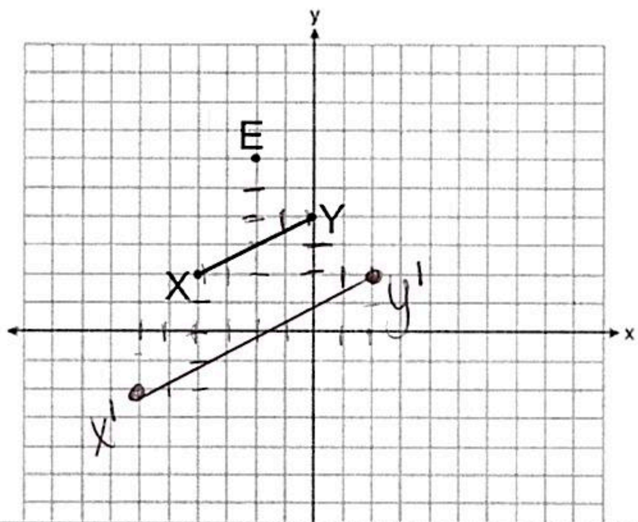
$3x = 100$

$x = \boxed{33.3 \text{ ft}}$

Dilating on a graph by center not at the origin

Count boxes (like slope) from center of dilation to one vertex, repeat same distances in same direction for dilation of 2! (continue to repeat again for dilation 3)
OR! Use a compass and straight edge!

15. On the graph below, point $E(-2,6)$ and $\overline{XY}$ with coordinates $X(-4,2)$ and $Y(0,4)$ are graphed. What are the coordinates of X' and Y' after $\overline{XY}$ undergoes a dilation centered at point E with a scale factor of 2?

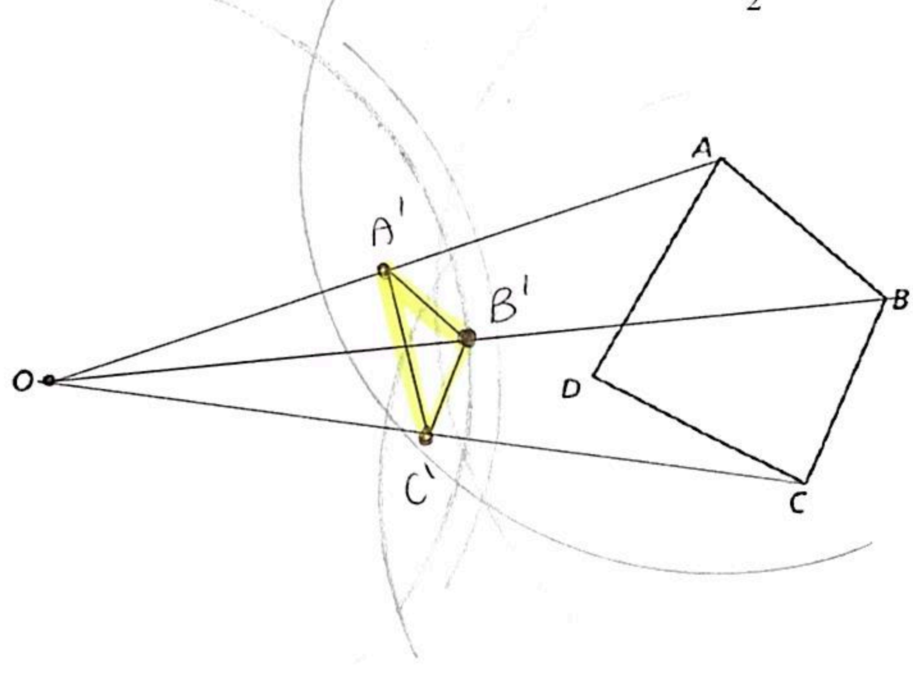


x' (6,-2)
y' (2,2)

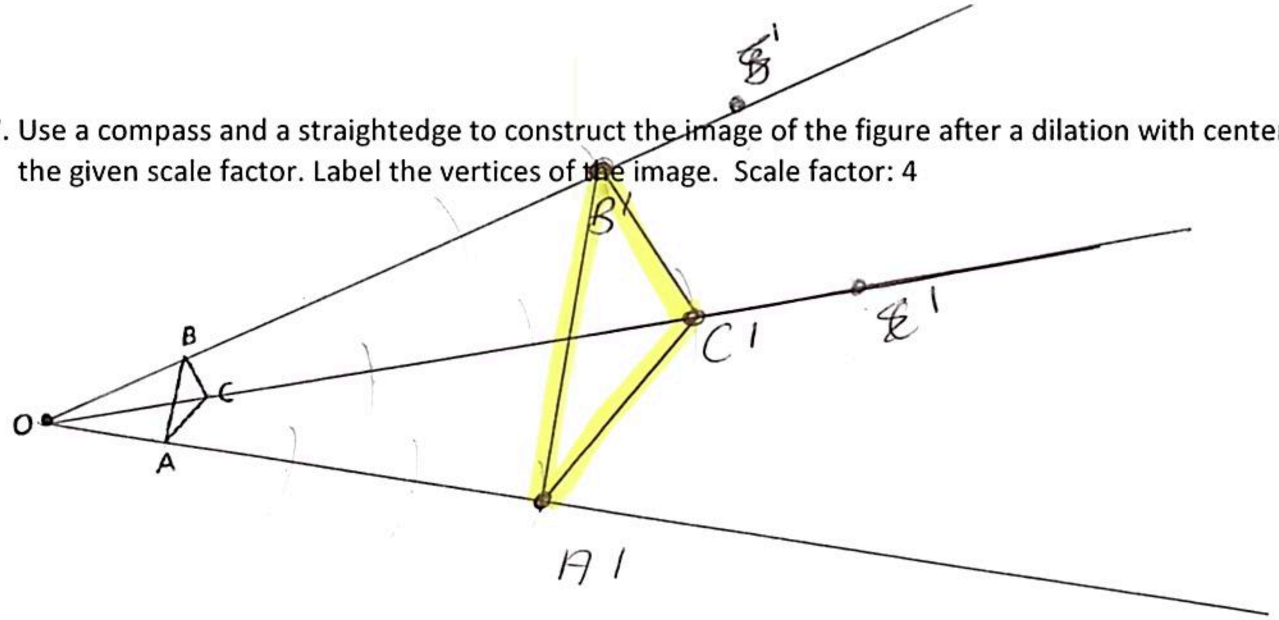
Constructing Dilations

- Use your compass to measure the distance between the center of dilation and a vertex. Expand the width indicated by the scale factor.
- If the scale factor is $\frac{1}{2}$, make a perpendicular bisector

16. Use a compass and a straightedge to construct the image of the figure after a dilation with center O and the given scale factor. Label the vertices of the image. Scale factor: $\frac{1}{2}$



17. Use a compass and a straightedge to construct the image of the figure after a dilation with center O and the given scale factor. Label the vertices of the image. Scale factor: 4

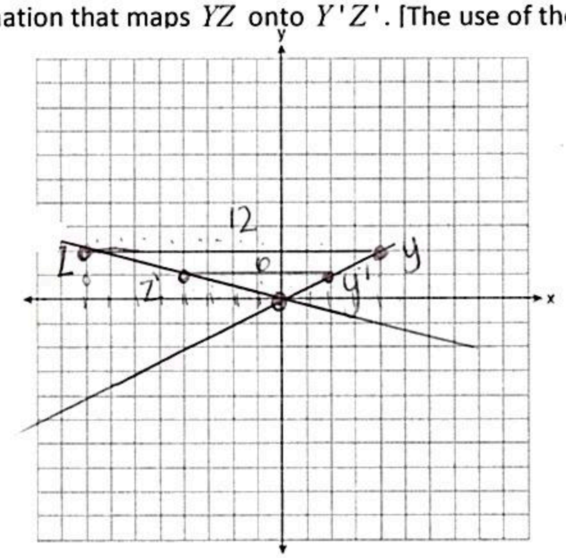


To find Center of Dilation	To Find Scale Factor
Connect Big to small (and PAST small) for 2 pairs of corresponding points.	$k = \frac{\text{new}}{\text{old}}$ (count lengths using only vertical/horizontal segments)

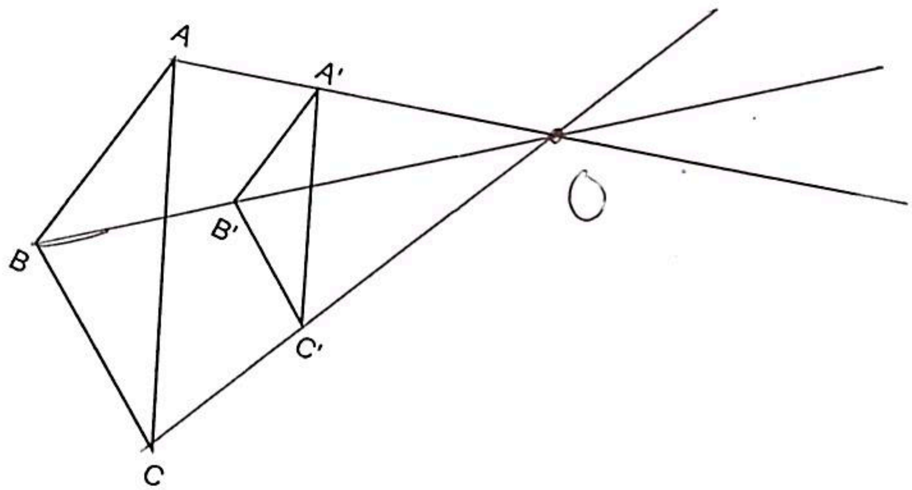
18. The coordinates of the endpoints of $\overline{YZ}$ are $Y(4,2)$ and $Z(-8,2)$. The coordinates of endpoints $\overline{Y'Z'}$ are $Y'(2,1)$ and $Z'(-4,1)$. Precisely describe the SINGLE transformation that maps $\overline{YZ}$ onto $\overline{Y'Z'}$. [The use of the set of axes below is optional.]

$K = \frac{new}{old} = \frac{6}{12} = \frac{1}{2}$

a dilation of $\frac{1}{2}$ centered at $(0,0)$



19. Locate the center of dilation that maps ΔABC to $\Delta A'B'C'$ and label it *O*.



To Dilate Centered at the Origin	In Similar Figures
Multiply each coordinate by the scale factor (k).	Corresponding sides are in proportion. Corresponding angles are ≅.

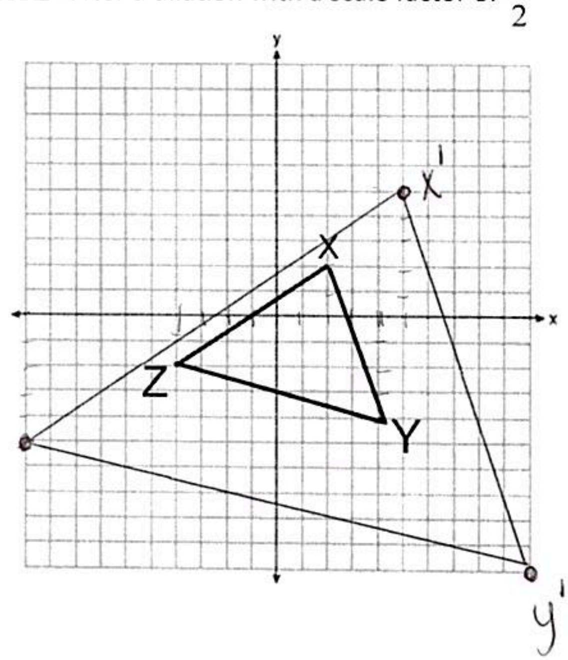
20. Triangle *XYZ* is graphed on the set of axes below.

a) On the same set of axes, graph and label $\Delta X'Y'Z'$, the image of ΔXYZ after a dilation with a scale factor of $\frac{5}{2}$ centered at the origin.

$X(2,2) \times \frac{5}{2} = X'(5,5)$

$Y(4,-4) \times \frac{5}{2} = Y'(10,-10)$

$Z(-4,-2) \times \frac{5}{2} = Z'(-10,-5)$



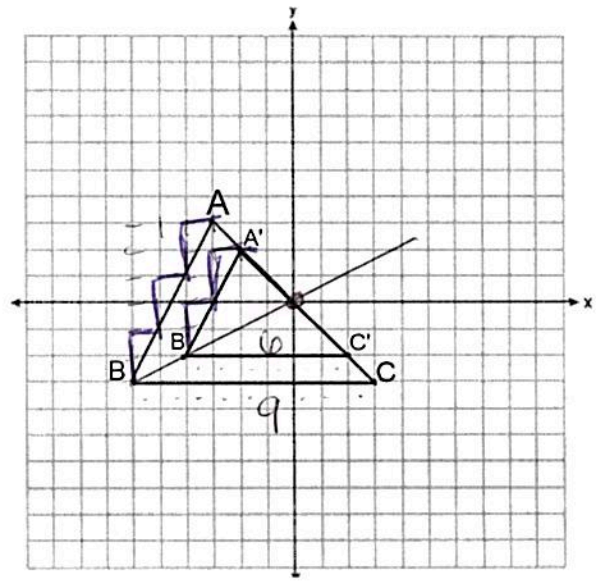
b) What is the relationship between the measure of $\angle X'Y'Z'$ and the measure of $\angle XYZ$?

$\angle X'Y'Z' \cong \angle XYZ$ b/c $\Delta XYZ \sim \Delta X'Y'Z'$ and similar Δ's have $\cong$ $\angle$'s

Describing Dilations	Finding Slope
Dilation needs • Center • Scale Factor	$\frac{rise}{run}$

21. In the diagram below, $\triangle A'B'C'$ is the image of $\triangle ABC$ after a SINGLE transformation.

a) Precisely describe the single transformation that was performed.



$K = \frac{new}{old} = \frac{6}{9} = \frac{1}{3}$

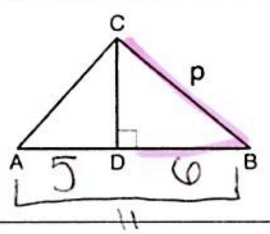
A dilation of $\frac{1}{3}$ centered @ (0,0)

b) Use slopes to explain why $\overline{A'B'} \parallel \overline{AB}$?

$\overline{A'B'} \parallel \overline{AB}$ b/c the slopes are both equal to $\frac{2}{1}$ (count boxes!)

HLLS	SAAS
$\frac{Hypotenuse}{Leg} = \frac{Leg}{Hypotenuse}$ *USE THIS IF THE ALTITUDE IS **NOT** LABELED!*	$\frac{Side_1}{Altitude} = \frac{Altitude}{Side_2}$ *USE THIS IF THE ALTITUDE IS LABELED!*

22. In the diagram below of triangle *ABC*, $\overline{CD}$ is the altitude to $\overline{AB}$, $CB = p$, AD = 5, and DB = 6. What value of p will make ΔABC a right triangle with a right angle at $\angle ACB$?



HLLS!

$\frac{11}{p} = \frac{p}{6}$

$\sqrt{p^2} = \sqrt{66}$

$\boxed{p = \sqrt{66}}$

23. Triangle *ABC* shown below is a right triangle with $\overline{AD}$ drawn to the hypotenuse $\overline{BC}$. If $BD = 2$ and $DC = 10$, what is the length of $\overline{AD}$ that makes $\overline{AD} \perp \overline{BC}$?

1) $2\sqrt{2}$
2) $2\sqrt{5}$
3) $2\sqrt{6}$
4) $2\sqrt{30}$

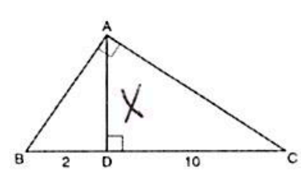
SAAS

$\frac{2}{x} = \frac{x}{10}$

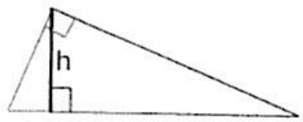
$\sqrt{20} = \sqrt{x^2}$

$\sqrt{4}\sqrt{5}$

$\boxed{x = 2\sqrt{5}}$



24. Find h:



SAAS!

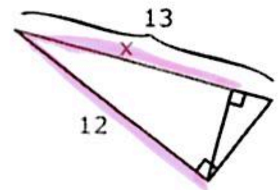
$$\frac{3}{h} = \frac{h}{9}$$

$$\sqrt{27} = \sqrt{h^2}$$

$$\sqrt{9}\sqrt{3}$$

$$\boxed{h = 3\sqrt{3}}$$

25. Find x: nearest tenth!



HLLS!

$$\frac{13}{12} = \frac{12}{x}$$

$$13x = 144$$

$$\boxed{x = 11.1}$$

26. In right triangle ABC shown in the diagram below, altitude $\overline{BD}$ is drawn to hypotenuse $\overline{AC}$, $CD = 12$, and $AD = 3$. What is the length of $\overline{AB}$? Leave your answer in simplest radical form.

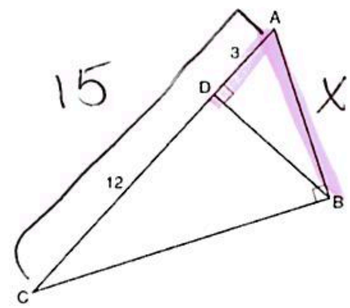
HLLS!

$$\frac{15}{x} = \frac{x}{3}$$

$$\sqrt{45} = \sqrt{x^2}$$

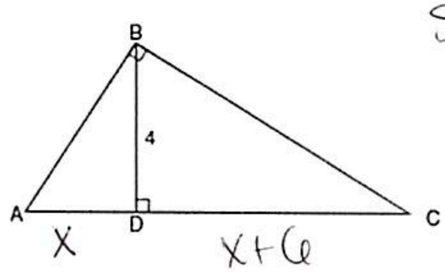
$$\sqrt{9}\sqrt{5}$$

$$\boxed{x = 3\sqrt{5}}$$



27. The drawing for a right triangular roof truss, represented by ΔABC, is shown in the accompanying diagram. If $\angle ABC$ is a right angle, altitude $BD = 4$ meters, and $\overline{DC}$ is 6 meters longer than $\overline{AD}$, find the length of base $\overline{AC}$ in meters.

x+6



SAAS!

$$\frac{x}{4} = \frac{4}{x+6}$$

FACTOR! set = 0

$$x(x+6) = 16$$

$$x^2 + 6x = 16$$

$$-16 \quad -16$$

$$x^2 + 6x - 16 = 0$$

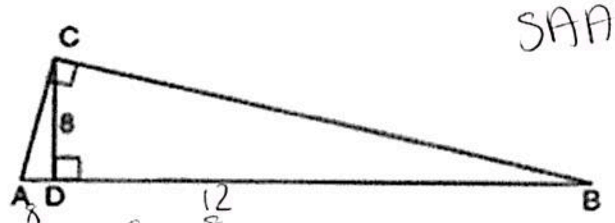
$$(x+8)(x-2) = 0$$

$x = -8$ reject | $\boxed{x = 2}$

$$AC = 2 + (2+6) = \boxed{10}$$

28. In the accompanying diagram, the altitude to the hypotenuse of right triangle ABC is 8. The altitude divides the hypotenuse into segments whose measures may be:

SAAS! Guess + check!

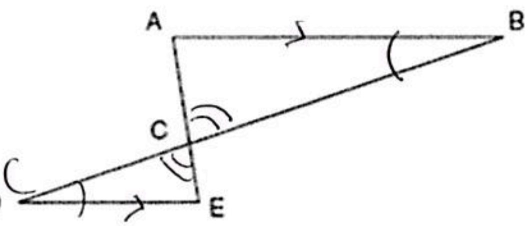


1) 8 and 12 $\quad \frac{8}{8} = \frac{8}{12}$ X
2) 3 and 24 $\quad \frac{3}{8} = \frac{8}{24}$ X
3) 6 and 10 $\quad \frac{6}{8} = \frac{8}{10}$ X
4) 2 and 32 $\quad \frac{2}{8} = \frac{8}{32}$ ✓

Similarity Proofs

	PROVE STATEMENT	REASON
1.	**Similarity Statement** $\Delta ABC \sim \Delta DEF$	$AA \cong AA$
2.	**Proportion** $\frac{AB}{BC} = \frac{DE}{EF}$	Corresponding parts of similar triangles are in proportion.
3.	**Product** $BCxDE = ABxEF$	The product of the means equals the product of the extremes

29. Given: $\overline{AE}$ and $\overline{DB}$ intersect at C
$\overline{AB} \parallel \overline{DE}$



Prove: $BC \cdot ED = AB \cdot DC$

$\frac{AB}{BC} = \frac{ED}{DC} \rightarrow \Delta ABC \sim \Delta EDC$

STATEMENT	REASON
① $\overline{AE}$ & $\overline{DB}$ intersect @C $\overline{AB} \parallel \overline{DE}$	① Given
② $\angle B \cong \angle D$ (A) ✓	② ∥ lines form ≅ alt. int. ∠
③ $\angle ACB \cong \angle ECD$	③ vertical ∠'s are ≅
④ $\Delta ABC \sim \Delta EDC$	④ AA ≅ AA
⑤ $\frac{AB}{BC} = \frac{ED}{DC}$	⑤ corresponding sides of ~ Δ's are in proportion
⑥ $BC \cdot ED = AB \cdot DC$	⑥ The product of the means equals the product of the extremes