Name: Key

Date:

UNIT 5

LESSON 12

AIM: THE PRODUCT OF THE MEANS EQUALS THE PRODUCT OF THE EXTREMES

Do Now: Simplify the following fractions

a) $\frac{2}{6} = \frac{1}{3}$
$6 = 6$ ✓

b) $\frac{5}{15} = \frac{1}{3}$
$15 = 15$ ✓

c) $\frac{10}{35} = \frac{2}{7}$
$70 = 70$ ✓

WHY can we do this?

extremes

means

$$\frac{1}{3} = \frac{2}{6}$$

$3 \cdot 2 = 1 \cdot 6$

$6 = 6$ ✓

- We solve proportions by cross multiplying. We can do this because the product of the means equals the product of the extremes!
- When triangles are similar, angles are congruent and sides are in proportion.
- Therefore, to prove triangles are similar, we need to state angles are congruent using AA.
- Once we have similar triangles we can say corresponding sides or similar triangles are in proportion.
- Finally, we can say the product of the means equals the product of the extremes!
- How will we know if our proof involves us stating the product of the means equals the product of the extremes? The prove statement will be a product!.

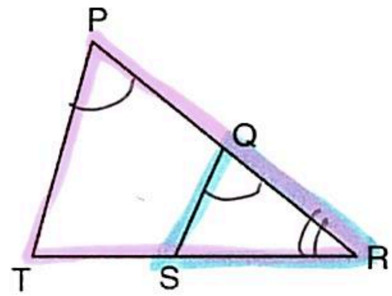
ORDER MATTERS!

	PROVE STATEMENT	REASON
1.	**Similarity Statement** $\Delta ABC \sim \Delta DEF$	$AA \cong AA$
2.	**Proportion** $\frac{AB}{BC} = \frac{DE}{EF}$	Corresponding parts of similar triangles are in proportion.
3.	**Product** $BCxDE = ABxEF$	The product of the means equals the product of the extremes

1. Given: Q is a point on $\overline{PR}$, S is a point on $\overline{TR}$, $\overline{QS}$ is drawn
   $\angle RPT \cong \angle RQS$

   means extremes

   (3) Prove: $PR \cdot RS = RT \cdot QR$
   1 2 1 2



What proportion can we set up that will give us this product?

(2) $\frac{RT}{PR} = \frac{RS}{QR}$ $\qquad \frac{ex._1}{mean_1} = \frac{mean_2}{ex._2}$

What triangles do we need to prove are similar first?

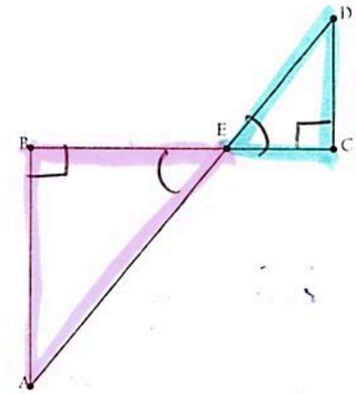
(1) $\triangle RTP \sim \triangle RSQ$

STATEMENT	REASON
(1) Q is a point on $\overline{PR}$, S is a point on $\overline{TR}$, $\overline{QS}$ is drawn $\angle RPT \cong \angle RQS$ (A) ✓	(1) Given
(2) $\angle R \cong \angle R$ (A) ✓	(2) Reflexive property
(3) $\triangle RTP \sim \triangle RSQ$	(3) $AA \cong AA$
(4) $\frac{RT}{PR} = \frac{RS}{QR}$	(4) Corresponding sides of $\sim \triangle$'s are in proportion
(5) $PR \cdot RS = RT \cdot QR$	(5) The product of the means = the product of the extremes

2. Given: $\overline{DC} \perp \overline{BC}$, $\overline{AB} \perp \overline{BC}$

Prove: $AB \cdot EC = EB \cdot DC$

(3) means extremes



What proportion can we set up that will give us this product?

(2) $\dfrac{EB}{AB} = \dfrac{EC}{DC}$ $\qquad \dfrac{ex._1}{mean_1} = \dfrac{mean_2}{ex._2}$

What triangles do we need to prove are similar first?

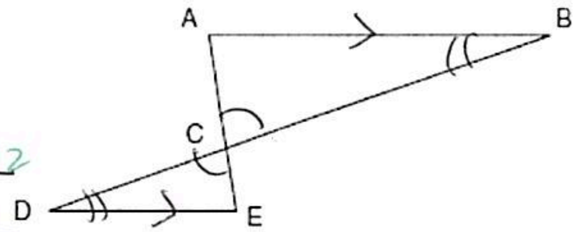
(1) $\triangle EBA \sim \triangle ECD$

STATEMENT	REASON
(1) $\overline{DC} \perp \overline{BC}$, $\overline{AB} \perp \overline{BC}$	(1) Given
(2) $\angle B \cong \angle C$ (A) ✓	(2) $\perp$ lines create $\cong$ right $\angle$'s
(3) $\angle BEA \cong \angle CED$ (A) ✓	(3) vertical $\angle$'s are $\cong$
(4) $\triangle EBA \sim \triangle ECD$	(4) $AA \cong AA$
(5) $\dfrac{EB}{AB} = \dfrac{EC}{DC}$	(5) corresponding sides of $\sim$ $\triangle$'s are in proportion
(6) $AB \cdot EC = EB \cdot DC$	(6) The product of the means equals the product of the extremes

3. Given: $\overline{AE}$ and $\overline{BD}$ intersect at C, and $\overline{AB} // \overline{ED}$

Prove: $AB \cdot DC = BC \cdot ED$

(3) 1 2 means 1 2 extremes



What proportion can we set up that will give us this product?

(2) $\frac{BC}{AB} = \frac{DC}{ED}$ $\qquad \frac{ex._1}{mean_1} = \frac{mean_2}{ex._2}$

What triangles do we need to prove are similar first?

(1) $\triangle BCA \sim \triangle DCE$

STATEMENT	REASON
(1) $\overline{AE}$ & $\overline{BD}$ intersect @ C $\overline{AB} \parallel \overline{ED}$	(1) Given
(2) $\angle ACB \cong \angle ECD$ (A) ✓	(2) vertical ∠'s are ≅
(3) $\angle D \cong \angle B$ (A) ✓	(3) ∥ lines form ≅ alt. int. ∠'s
(4) $\triangle BCA \sim \triangle DCE$	(4) AA ≅ AA
(5) $\frac{BC}{AB} = \frac{DC}{ED}$	(5) corresponding sides of ~ Δ's are in proportion
(6) $AB \cdot DC = BC \cdot ED$	(6) The product of the means equals the product of the extremes

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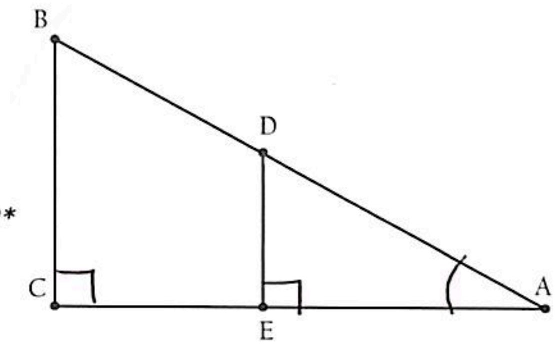
LESSON 12

HOMEWORK

1. For the following, fill in the missing pieces.

PRODUCT	$ER \cdot SD = TE \cdot DT$	$ED \cdot AB = AD \cdot CB$	$EB \cdot DC = AB \cdot EC$	$RT \cdot QR = PR \cdot RS$
PROPORTION	$\frac{TE}{ER} = \frac{SD}{DT}$	$\frac{AD}{ED} = \frac{AB}{CB}$	$\frac{AB}{EB} = \frac{DC}{EC}$	$\frac{PR}{RT} = \frac{QR}{RS}$
SIMILARITY STATEMENT	$\Delta TER \sim \Delta SDT$	$\Delta ADE \sim \Delta ABC$	$\Delta ABE \sim \Delta DCE$	$\Delta PRT \sim \Delta QRS$

2. Given: In right triangle ABC, $\angle C = 90°$, $\overline{DE} \perp \overline{AC}$ Fix!



Prove: $\cancel{\frac{AD}{ED} = \frac{AB}{CB}}$ $ED \cdot AB = AD \cdot CB$

What proportion can we set up that will give us this product?

$\frac{AD}{ED} = \frac{AB}{CB}$

What triangles do we need to prove are similar first?

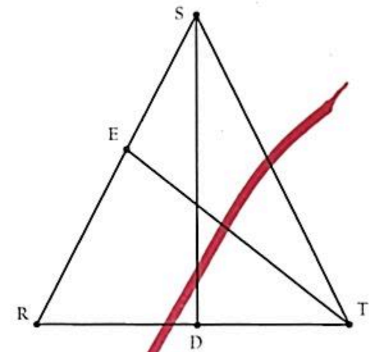
$\Delta ADE \sim \Delta ABC$

STATEMENT	**REASON**
① Right ΔABC, $\angle C = 90°$, $\overline{DE} \perp \overline{AC}$	① Given
② $\angle AED$ is a right $\angle$	② $\perp$ lines form right $\angle$'s
③ $\angle C \cong \angle AED$ (A) ✓	③ Right $\angle$'s are $\cong$
④ $\angle A \cong \angle A$ (A) ✓	④ Reflexive Property
⑤ $\Delta ADE \sim \Delta ABC$	⑤ AA $\cong$ AA
⑥ $\frac{AD}{ED} = \frac{AB}{CB}$	⑥ corresponding sides of $\sim \Delta$'s are in proportion
⑦ $ED \cdot AB = AD \cdot CB$	⑦ The product of the means equals the prod. of the extremes

4. Given: ΔSRT with $\overline{SR} \cong \overline{ST}$

$\overline{TE} \perp \overline{RS}$, $\overline{SD} \perp \overline{RT}$

Prove: $ER \cdot SD = TE \cdot DT$



What proportion can we set up that will give us this product?

What triangles do we need to prove are similar first?

STATEMENT	REASON