

**LESSON #9: THE REMAINDER THEOREM & FINDING A REMAINDER ON A GRAPH**

**Do Now:** Consider the polynomial function  $g(x) = x^3 - 3x^2 + 6x + 8$ .

a. Divide  $g$  by  $x + 1$ .

$$\begin{array}{r}
 x^2 - 4x + 10 + \frac{2}{x+1} \\
 x+1 \overline{) x^3 - 3x^2 + 6x + 8} \\
 \underline{-(x^3 + x^2)} \phantom{+ 6x + 8} \\
 -4x^2 + 6x \phantom{+ 8} \\
 \underline{+4x^2 + 4x} \phantom{+ 8} \\
 10x + 8 \\
 \underline{-(10x + 10)} \\
 -2
 \end{array}$$

b. Find  $g(-1)$ .

$$\begin{aligned}
 g(-1) &= (-1)^3 - 3(-1)^2 + 6(-1) + 8 \\
 &= -1 - 3 - 6 + 8 \\
 &= \boxed{-2}
 \end{aligned}$$

← same!

The remainder found after dividing  $P$  by  $x - a$  will be the same value as  $P(a)$

If  $p(a) = 0$ , then  $(x - a)$  is a factor

What do we suspect about the connection between the remainder from dividing a polynomial  $P$  by  $x - a$  and the value of  $P(a)$ ?

1) Consider the polynomial:  $P(x) = x^4 + 3x^3 - 28x^2 - 36x + 144$ . Is  $x + 3$  one of the factors of  $P$ ?

↑  
Factor
↑  
Root

$$\begin{aligned}
 P(-3) &= (-3)^4 + 3(-3)^3 - 28(-3)^2 - 36(-3) + 144 \\
 &= 81 - 81 - 252 + 108 + 144 \\
 &= 0
 \end{aligned}$$

Root = -3

yes  $x + 3$  is a factor. b/c it satisfies the Poly. remainder

2) When  $x^3 + kx^2 - 4x + 2$  is divided by  $x + 2$ , the remainder is 26. Find  $k$ .

Root = -2

$$\begin{aligned}
 P(-2) &= 26 \\
 (-2)^3 + k(-2)^2 - 4(-2) + 2 &= 26 \\
 -8 + 4k + 8 + 2 &= 26 \\
 4k + 2 &= 26 \\
 4k &= 24 \\
 \boxed{k = 6}
 \end{aligned}$$

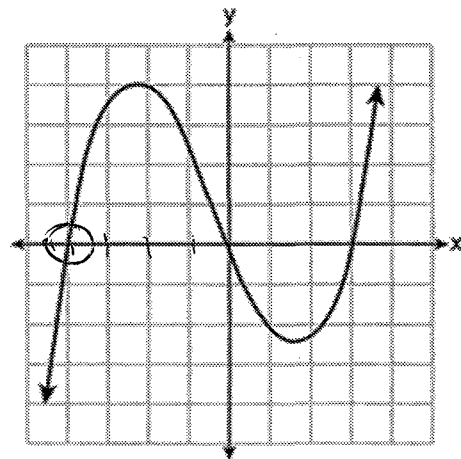
3) The graph of  $p(x)$  is shown in the accompanying diagram. What is the remainder when  $p(x)$  is divided by  $x+4$ ?  $\rightarrow$  ROOT = -4

1)  $x-4$

2)  $-4$

3) 0  $-4$  is a root so there is no remainder!

4) 4

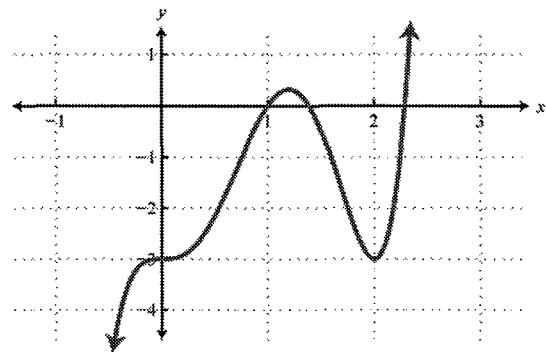


4) The graph of a polynomial function is illustrated below. What is the remainder when  $p(x)$  is divided by  $(x-2)$ ? ROOT = 2  $-3$ ?

Find  $P(2)$

in other words, find the y-value when  $x=2$

$P(2) = \boxed{-3} \rightarrow$  remainder!



5) Consider the polynomial function:  $P(x) = x^3 - 8x^2 - 29x + 180$ . If  $P(9) = 0$ , find the remaining two factors of  $P$ .  $\rightarrow$  means  $x-9$  is a factor

①

$$\begin{array}{r} x^2 + x - 20 \\ x-9 \overline{) x^3 - 8x^2 - 29x + 180} \\ \underline{-x^3 + 9x^2} \phantom{+ 180} \\ 1x^2 - 29x \phantom{+ 180} \\ \underline{-x^2 + 9x} \phantom{+ 180} \\ -20x + 180 \\ \underline{+20x - 180} \\ 0 \end{array}$$

②  $x^2 + x - 20 = (x+5)(x-4)$

$$\{(x+5)(x-4)(x-9)\}$$

6) If  $p(a)$  is the remainder when  $x^3 + 3x^2 - 18x - 40$  is divided by  $x-a$ , for which value of  $a$  would  $p(a) = 0$ ? GUESS & CHECK OR FACTOR OR CALCULATOR

(A) 2      (B) -2      (C) 3 (FASTEST)      (D) -3

Practice:

- 7) Use the Remainder Theorem to find the remainder for the following division.

$$(k^3 - k^2 - k - 2) \div (k - 2)$$

$$P(2) = (2)^3 - (2)^2 - (2) - 2$$

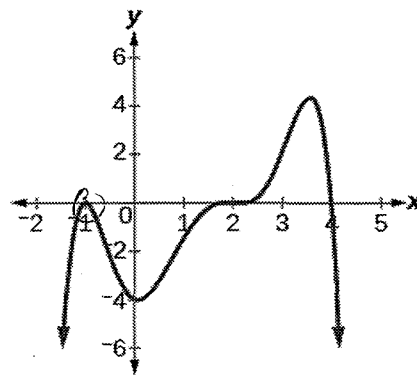
$$8 - 4 - 2 - 2 = 0$$

$$\boxed{\text{Remainder} = 0}$$

- 8) The graph of a polynomial function,  $M(x)$ , is illustrated below. What is the remainder when  $M(x)$  is divided by  $x+1$ ?  $-1 = \text{root}$

$$P(-1) = 0$$

$$\boxed{\text{Remainder} = 0}$$



- 9) Find  $a$  such that  $2x+5$  will be a factor of  $4x^3 + 8x^2 + ax + 30$ .

$$2x + 5 = 0$$

$$\begin{array}{r} -5 \quad -5 \\ \hline 2x = -5 \end{array}$$

$$2x = -5$$

$$x = \frac{-5}{2} \text{ (root)}$$

$$\text{want } P\left(-\frac{5}{2}\right) = 0 \quad \text{OR} \quad P(-2.5) = 0$$

$$4(-2.5)^3 + 8(-2.5)^2 + a(-2.5) + 30 = 0$$

$$-62.5 + 50 - 2.5a + 30 = 0$$

$$17.5 - 2.5a = 0$$

$$-2.5a = -17.5$$

$$\boxed{a = -7}$$

- 10) Determine if  $x-5$  is a factor of  $2x^3 - 4x^2 - 7x - 10$ . Explain your answer.

$$\text{root} = 5$$

$$P(5) = 2(5)^3 - 4(5)^2 - 7(5) - 10$$

$$= 250 - 100 - 35 - 10$$

$$= \boxed{105}$$

No,  $x-5$  is not a factor b/c it has a remainder of 105.

