

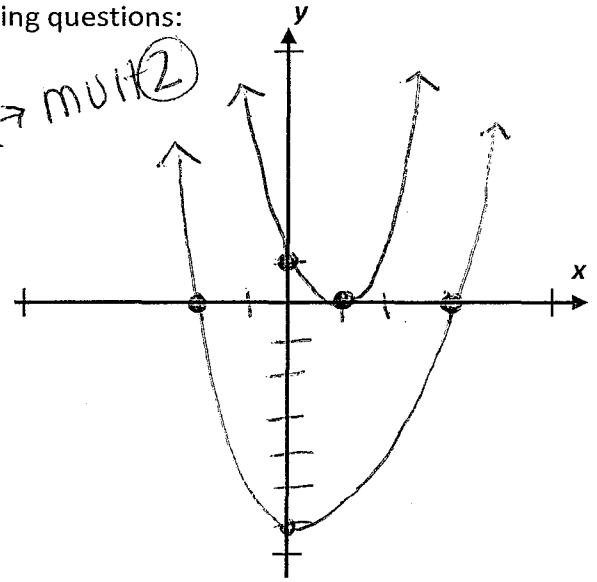
LESSON #1: MULTIPLICITY

Do Now: Given the following quadratic equations, answer the following questions:

Mult 1
 I. $x^2 - x - 6 = 0$
 $(x-3)(x+2)$
 $x=3$ | $x=-2$ → *Mult 1*

II. $x^2 - 2x + 1 = 0$
 $(x-1)(x-1)$
 $x=1$ | $x=1$ → *Mult 2*

- a) What is the degree of each polynomial?
 2
- b) How many roots will you expect there to be?
 2
- c) What is the leading coefficient of the equations?
 1



- d) Find the roots of the quadratic equations algebraically and sketch without the use of a calculator.
 ↳ 2 roots + y intercept

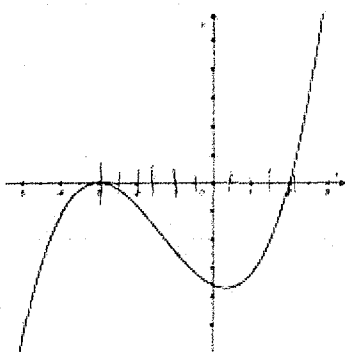
WORD	DEFINITION
Multiplicity	The # of times a polynomial function has a root at a given point

- When the multiplicity is EVEN, the graph will bounce at the root.
- When the multiplicity is ODD, the graph will pass through at the root.

For questions #1-2,

- a) Find the zeros of the following polynomial functions.
 b) State their multiplicities
 c) State the degree of the polynomial.
 d) Write the equation in factored form.

1)

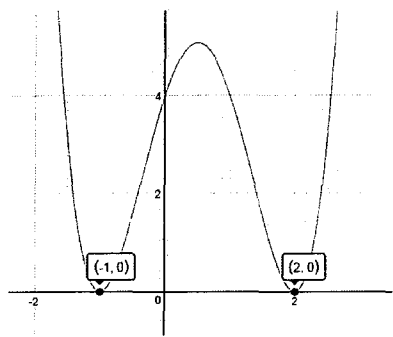


→ don't mult it out

- a) -5, mult 2
 4, mult 1
 b) degree = 3

c) $y = (x+5)(x+5)(x-4)$
 or
 $y = (x+5)^2(x-4)$

2)



a) $-1 \text{ mult } +2$
 $2 \text{ mult } +2$

b) Degree = 4

c) $y = (x+1)^2(x-2)^2$

3) State a polynomial that has the roots given {4, -5, 0} and 1 as the leading coefficient.

$y = (x-4)(x+5)(x+0)$

$y = x^3 + 5x^2 - 4x^2 - 20x$

$y = x(x-4)(x+5)$

$y = (x^2 - 4x)(x+5)$

$y = x^3 + x^2 - 20x$

PRACTICE:

4) Suppose we know that the polynomial equation has three real solutions and that one of the factors of $4x^3 - 12x^2 + 3x + 5 = 0$ is $(x-1)$. Find all the solutions to the given equation.

①
$$\begin{array}{r} 4x^2 - 8x - 5 \\ x - 1 \overline{) 4x^3 - 12x^2 + 3x + 5} \\ \underline{-4x^3 + 4x^2} \\ -8x^2 + 3x \\ \underline{+8x^2 - 8x} \\ -5x + 5 \\ \underline{+5x - 5} \\ 0 \end{array}$$

② $4x^2 - 8x - 5$

$4x^2 + 2x - 10x - 5$

$2x(2x+1) - 5(2x+1)$

$(2x-5)(2x+1)$

$x = \frac{5}{2}$ $x = -\frac{1}{2}$

$\left\{ -\frac{1}{2}, 1, \frac{5}{2} \right\}$

5) Given the accompanying graph:

a) Find the degree of the equation.

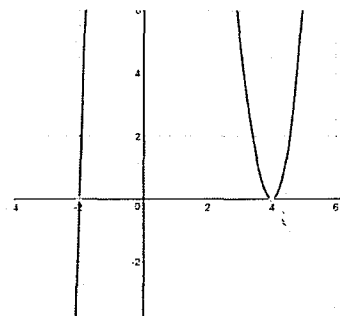
3

b) Find the zeroes of the graph.

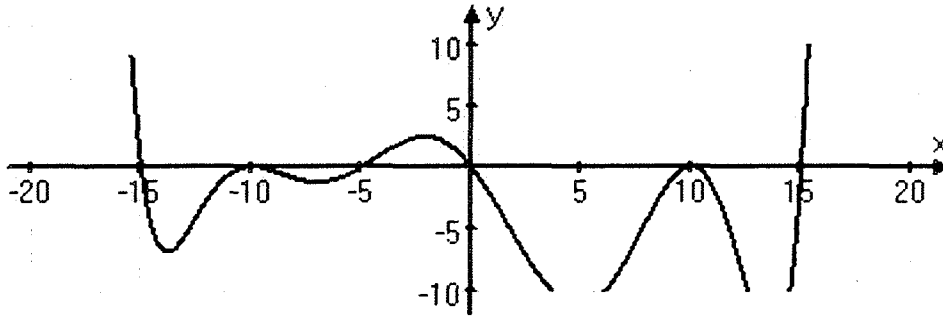
$-2, 4 \text{ mult } +2$

c) Identify the factors.

$(x+2)(x-4)^2$



6) The following graph shows an eighth-degree polynomial. List the polynomial's zeroes with their multiplicities (even or odd).



Zeros	Multiplicity
-15	odd
-10	even
-5	odd
0	odd
10	even
15	odd

7) Extra Practice: Go back to question #5, and write the equation in standard form.

$$y = (x-4)^2(x+2)$$

$$y = (x-4)(x-4)(x+2)$$

$$x^2 - 4x - 4x + 16$$

$$(x^2 - 8x + 16)(x+2)$$

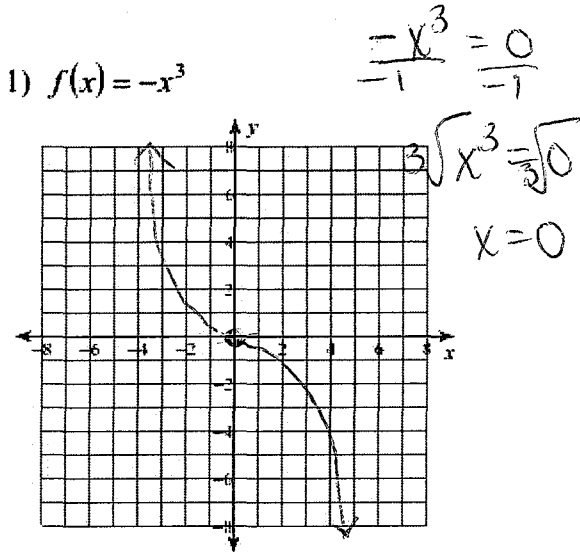
	x^2	$-8x$	$+16$
x	x^3	$-8x^2$	$+16x$
$+2$	$+2x^2$	$-16x$	32

$$y = x^3 - 6x^2 + 32$$

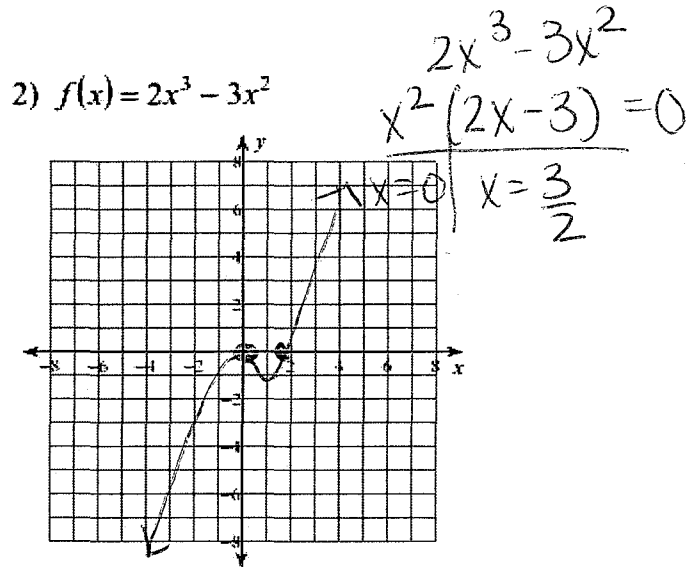
LAB #4

For each of the following polynomial functions:

- State the degree of the function
- State all zeroes and their multiplicities
- Sketch each graph.

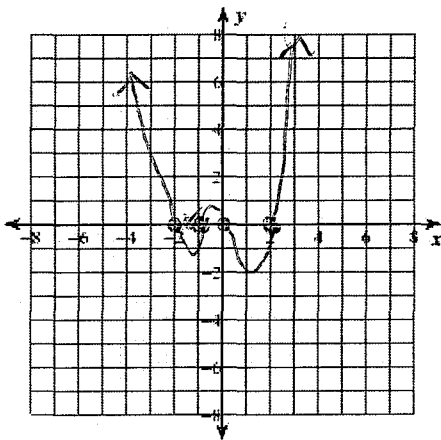


- deg = 3
- 0 mult + 3



- deg = 3
- $\frac{3}{2}$ mult + 1, 0 mult + 2

3) $f(x) = x^4 + x^3 - 4x^2 - 4x$



- deg = 4
- $$x^4 + x^3 - 4x^2 - 4x$$

$$x^3(x+1) - 4x(x+1)$$

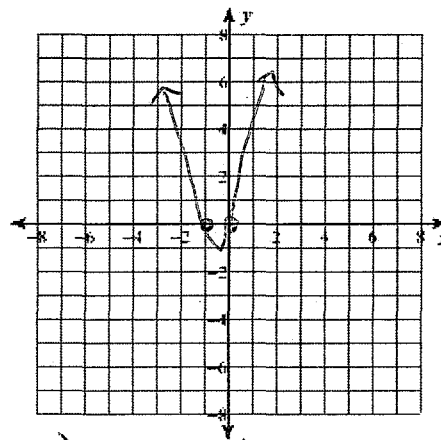
$$(x^3 - 4x)(x+1)$$

$$x(x^2 - 4)(x+1)$$

$$x(x+2)(x-2)(x+1)$$

0 | -2 | 2 | -1 } mult + 1

4) $f(x) = x^4 + x^3$



- deg = 4
- $$x^4 + x^3$$

$$\frac{x^3(x+1)}{x^3} = \frac{0}{x^3}$$

$x = 0$ | $x = -1$
 mult + 3 | mult + 1

5) Find all zeroes of the following functions with their multiplicities and state the degree:

$f(x) = \frac{(x-4)(x+4)^3}{4 \mid -4 \text{ mult } 3}$ <p>degree = 4</p>	$f(x) = \frac{(x-3)(x+1)}{3 \mid -1}$ <p>degree = 2</p>
$f(x) = \frac{(x-1)^2 \cdot (x+3)^5}{1 \mid -3}$ <p>mult + 2 mult + 5</p> <p>degree = 7</p>	$f(x) = \frac{x(x-2)(x+1)}{x=0 \mid 2 \mid -1}$ <p>degree = 3</p>

6) Suppose one of the factors of $x^3 - 10x^2 + 27x - 18$ is $(x-3)$, what are the other two factors?

①

$$\begin{array}{r} x^2 - 7x + 6 \\ x-3 \overline{) x^3 - 10x^2 + 27x - 18} \\ \underline{-x^3 + 3x^2} \downarrow \\ -7x^2 + 27x \downarrow \\ \underline{+7x - 21x} \downarrow \\ 6x - 18 \\ \underline{-6x + 18} \\ 0 \end{array}$$

② $x^2 - 7x + 6$
 $(x-6)(x-1)$

③ $\{(x-3)(x-6)(x-1)\}$

7) Suppose we know that the polynomial equation has three real solutions and that one of the roots of

$x^3 + 3x^2 - 4x - 12 = 0$ is $x = -3$. State all solutions.

①

$$\begin{array}{r} x^2 + 0x - 4 \\ x+3 \overline{) x^3 + 3x^2 - 4x - 12} \\ \underline{-x^3 + 3x^2} \downarrow \\ 0x^2 - 4x \downarrow \\ \underline{-0x^2 + 0x} \downarrow \\ -4x - 12 \\ \underline{+4x + 12} \\ 0 \end{array}$$

② $x^2 - 4$
 $(x+2)(x-2)$
 $-2 \mid 2$

③ $\{-3, -2, 2\}$

SOAPI!

8) Factor: $125x^3 - 27$

$$\sqrt[3]{125x^3} = 5x$$

$$\boxed{(5x-3)(25x^2+15x+9)}$$

$$\sqrt[3]{27} = 3$$

9) Factor: $2x^2 - 8x + 12$

$$2x^2 - 8x + 3x - 12$$

$$2x(x-4) + 3(x-4)$$

$$\boxed{(2x+3)(x-4)}$$

10) Factor: $k^4 - 4k^2 + 8k^3 - 32k + 12k^2 - 48$

$$k^2(k^2-4) + 8k(k^2-4) + 12(k^2-4)$$

$$(k^2 + 8k + 12)(k^2 - 4)$$

$$\boxed{(k+6)(k+2)(k+2)(k-2)}$$