Name: $\qquad$
REGENTS REVIEW

Date: $\qquad$
BATTA

## MODULE 1 REVIEW

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| Topic A: <br> Basic Constructions | - Construct an Equilateral Triangle, <br> - Copy and Bisect an Angle, <br> - Construct a Perpendicular Bisector, <br> - Points of Concurrencies | $\begin{aligned} & \text { G-CO.1, } \\ & \text { G-CO.12, } \\ & \text { G-CO. } \end{aligned}$ | 2-8 |
| Topic B: <br> Unknown Angles | - Solving for Unknown angles, <br> - Parallel lines and transversals, <br> - Exterior angle theorem, <br> - Auxiliary lines, <br> - proving known facts | G. CO-9 | 9-13 |
| $\begin{gathered} \text { Topic C: } \\ \text { Transformations/ } \\ \text { Rigid Motions } \end{gathered}$ | - Rotations, reflections, translations, <br> - Symmetry, <br> - Sequence of rigid motions, <br> - Transformations on the coordinate plane. | $\begin{gathered} \text { G-CO.2-G-CO. } 7 \\ \text { G-CO. } 12 \end{gathered}$ | 14-19 |
| Topic D: <br> Congruence | - Congruence Criteria-SAS, ASA, SSS, SAA and HL, <br> - Proving triangles congruent, <br> - Isosceles triangles, <br> - Congruence in terms of rigid motions, <br> - Corresponding parts of $\cong$ triangles. | $\begin{aligned} & \text { G-CO.7, } \\ & \text { G-CO. } \end{aligned}$ | 20-24 |
| Topic E: <br> Proving Properties of Geometric Figures | - Properties of parallelograms, <br> - Parallelogram proofs, <br> - Mid-segment of a triangle, <br> - Centroid of a triangle. | $\begin{aligned} & \text { G-CO.9, } \\ & \text { G-CO.10, } \\ & \text { G-CO. } 11 \end{aligned}$ | 25-28 |
|  | MIXED REVIEW |  | 29-32 |

## Topic A- Vocabulary and Constructions

Example 1: Fill in the puzzle below using the vocabulary listed in the word bank.


## ACROSS

3. An angle measuring more than 90 and less than 180 degrees
4. A part of a line starting at one endpoint and going on forever through the other point on the line
5. Two noncollinear rays with a common endpoint form an $\qquad$
6. A triangle with all sides and all angles congruent
7. A point that divides a line segment into two congruent halves
8. An angle less than 90 degrees
9. Points that lie on the same line
10. Lines that form a right angle
11. An angle measuring 180 degrees
12. A set of instructions for drawing points, lines, circles and figures in a plane

Word Bank:
Collinear
Obtuse
Midpoint
Perpendicular
Construction
Equilateral

## DOWN

1. A figure with a center point and all points the same distance away from the center
2. Point $B$ is said to be $\qquad$ from A and C if $A B=B C$
3. A part of a line between two endpoints
4. The distance from the center of the circle to any point on the circumference
5. A ray that divides an angle into two congruent parts
6. A triangle with two equal legs and two equal base angles

## Constructions

Example 2: Using a straightedge and a compass, construct the following:
a)Equilateral Triangle

c) Copy an Angle


| Term | Definition |
| :---: | :---: |
| Median | A segment drawn from one vertex of a triangle to the ___ of the opposite side. |
| Altitude | A segment drawn from one vertex of a triangle $\quad . \quad$ to opposite side. |

Example 3: In the diagram of $\triangle A B C$ shown below, use a compass and straightedge to construct the following:
a) Median from C to $\overline{A B}$.

b) Altitude from B to $\overline{A C}$


| Term | Definition |  |
| :---: | :--- | :---: |
| Circumcenter | Point of concurrency of $3 \ldots$ | in a triangle. |
| Incenter | Point of concurrency of 3__ in a triangle. |  |

Example 4: a) Construct the circumcenter of $\triangle R S Q$ and label it 0.

b) Construct the incenter of $\triangle D E F$ and label it 0 .


Example 5: Using a compass and a straightedge, on the line shown, construct $\Delta X^{\prime} Y^{\prime} Z^{\prime}$, such that $\Delta X^{\prime} Y^{\prime} Z^{\prime} \cong \Delta \mathrm{XYZ}$. [Leave all construction marks]


## Mixed Practice with Topic A Pages 5-8

1. Identify the construction we have covered matches each diagram.

Diagram 1
Diagram 2


2. A student has done the following construction. What was this student attempting to construct? Is there more than one thing that the student could be constructing? Explain.

3. Which diagram represents a correct construction of equilateral $\triangle A B C$, given side $\overline{A B}$ ?

(1)

(2)

(3)

(4)
4. Which construction is completed in the diagram below to create parallel lines?

1) The angle bisector of $\angle A B L$
2) A perpendicular line $\overline{B A}$
3) The perpendicular bisector of $\overline{B C}$
4) Copying $\angle A B L$

5. Use the diagram to complete the relationship. (The compass was constant for each individual construction.)

a) $\mathrm{DF}=$ $\qquad$
b) $\mathrm{CE}=$ $\qquad$
c) $m \angle A B F=$ $\qquad$
6. A teacher finds a paper on the ground in the classroom. When she looks at it carefully she realizes it is from her geometry class because it has a construction on it. Which of the following constructions is NOT FOUND directly from this student's work?
1) The midpoint of $\overline{A B}$
2) The perpendicular bisector of $\overline{A B}$
3) A perpendicular line to $\begin{array}{ll}\overline{A B} & \text { 4) The angle bisector of } \angle \mathrm{CAB}\end{array}$
入c


7. Which construction is represented by these construction marks?
1) Copying $\angle A B C$
2) The perpendicular bisector of $\overline{B C}$

8. The diagram below shows the construction of the center of the circle circumscribed about $\triangle A B C$. This construction represents how to find the intersection of
1) the angle bisectors of $\triangle A B C$
2) the medians to the sides of $\triangle A B C$
3) the altitudes to the sides of $\triangle A B C$
4) the perpendicular bisectors of the sides of $\triangle A B C$

9. Which geometric principle is used in the construction shown below?
1) The intersection of the angle bisectors of a triangle is the center of the inscribed circle
2) The intersection of the angle bisectors of a triangle is the center of the circumscribed circle
3) The intersection of the perpendicular bisectors of the sides of a triangle is the center of the inscribed circle.
4) The intersection of the perpendicular bisectors of the sides of a triangle is the center of the circumscribed circle

10. In the diagram below, it is given that $\overline{R S}$ bisects $\overline{M N}$ at point P . Which of the following statements does not have to be true?
1) $M P=N P$
2) $R P+P S=R S$
3) $P$ is the midpoint of $\overline{R S}$
4) P is the midpoint of $\overline{M N}$

11. Triangle $X Y Z$ is shown below. Using a compass and straightedge, on the line below, construct and label $\triangle A B C$, such that $\triangle A B C \cong \triangle X Y Z$. [Leave all construction marks.]

12. Using a compass and straightedge, construct the circumcenter of $\triangle A R T$ shown below and label it $O$.
13. Construct the angle bisector of $\angle \mathrm{ABC}$.

14. Using a compass and a straightedge, construct the perpendicular bisector of the $\overline{B V}$. Label it $\overleftrightarrow{A X}$.

b) Identify the midpoint of the line segment with the letter $M$.
c) What can you say about any point on $\overleftrightarrow{A X}$ in relation to endpoints B and V.
15. Jo wants to open a new soft drink stand at an amusement park. She looks at the map of the amusement park and notes the 2 most popular rides: roller coaster, and swings. Jo decides to locate the stand so that it is the same distance from these 2 rides. Why do you think that Jo wants the stand located at a point equidistant from the 2 most popular rides? Indicate where the stand should be placed with the letter X.

16. Right angle, $\angle X Y Z$, was bisected below. What is the measure of $\angle A Y Z$ ? Explain how you came to your solution.

17. If the length of $\overline{A B}$ is 12 cm , what is the length of $\overline{A H}$ after the construction was performed below?


## Topic B- Important Geometry Facts and Theorems

Fill in the "Fact/Discovery" column based on geometry facts you have learned!

| Types of Angles | Diagram | Fact/Discovery |
| :---: | :---: | :---: |
| Vertical Angles |  |  |
|  <br> Supplementary Angles |  |  |
| Adjacent Angles on a Line |  |  |
| Angles around a Point |  |  |

Corresponding Angles

Fill in the "Fact/Discovery" column based on geometry facts you have learned!

| Sum of | Diagram | Fact/Discovery |
| :--- | :--- | :--- |
| Isosceles |  |  |
|  |  |  |


| Exterior of a <br> Theorem | $\bigwedge_{B}^{A}$ |  |
| :--- | :--- | :--- |

Example 1: In the diagram below, $\overline{E F} \| \overline{D C}, m \angle A B C=112^{\circ}$ and $m \angle E A D=24^{\circ}$. What is the measure of $\angle D A B$ ?


Example 2: Use the diagram to determine the answers.
a) $\angle 2$ and $\angle 4$ are vertical angles.
b) $\angle 1, \angle 2$, and $\angle 3$ add to $180^{\circ}$.
c) $\angle 1$ and $\angle 6$ are complementary.
T or F
d) $\angle 3$ and $\angle 4$ are adjacent angles.
T or F
T or F
T or F


Example 3: In the accompanying diagram of $\triangle B C D, \triangle A B C$ is an equilateral triangle and $A D=A B$. What is the value of $x$, in degrees? Explain how you reached your solution.


Example 4: In the diagram of $\triangle A B C$ below, $\overline{A B}$ is extended to point $D$. If $\mathrm{m} \angle C A B=x+40, \mathrm{~m} \angle A C B=3 x+10$, $\mathrm{m} \angle C B D=6 x$, what is $\mathrm{m} \angle C A B$ ?


Example 5: Fill in the missing reasons for steps 2 and 3 to prove the sum of the angles of a triangle is $180^{\circ}$.
Given: $\triangle A B C$
Prove: $\mathrm{m} \angle 1+\mathrm{m} \angle 2+\mathrm{m} \angle 3=180^{\circ}$


Example 6: Fill in the missing reasons for steps 2 and 3 to prove that the exterior angle is equal to the sum of the triangles two remote angles.

Given: $\triangle A B C$ with external angle, $\angle A C D$.
Prove: $m \angle A C D=m \angle B+m \angle A$


## STATEMENT

1) $\triangle A B C$ with external angle, $\angle A C D$.
2) $m \angle A C B+m \angle B+m \angle A=180^{\circ}$
3) $m \angle A C B+m \angle A C D=180^{\circ}$
4) $m \angle A C B+m \angle A C D=m \angle A C B+m \angle B+m \angle A$
5) $m \angle A C D=m \angle B+m \angle A$

## REASON

1) Given
2) 
3) 
4) Substitution Property(both $=180^{\circ}$ so must $=$ each other)
5) Subtraction Property (subtracted $\mathrm{m} \angle \mathrm{ACB}$ both sides)

## Mixed Practice with Topic B Pages 12-13

1. In the following diagram $g \| h$. State an angle that is congruent to $\angle 7$ and explain why.

2. Based on the diagram, which statement is true?
1) $a \| b$
2) $a \| c$
3) $b \| c$
4) $d \| e$

3. In the diagram below, $\triangle L M O$ is isosceles with $L O=M O$. If $\mathrm{m} \angle L=55$ and $\mathrm{m} \angle N O M=28$, what is $\mathrm{m} \angle N$ ? Explain your solution.

4. In the diagram below, lines $p$ and $s$ are cut by transversals. The angles are marked as shown. Explain why $p$ and $s$ must be parallel.

5. If the measures of the angles of a triangle are represented by $2 x, 3 x-15$, and $7 x+15$, the triangle is
1) an isosceles triangle
2) a right triangle
3) an acute triangle
4) an equiangular triangle
6. In the diagram below, $\overleftrightarrow{F E}$ bisects $\overline{A C}$ at B , and $\overleftrightarrow{G E}$ bisects $\overline{B D}$ at C . Which statement is always true?
(1) $\overline{A B} \cong \overline{D C}$
(3) $\overrightarrow{B D}$ bisects $\overrightarrow{G E}$ at C .
(2) $\overline{F B} \cong \overline{E B}$
(4) $\overleftrightarrow{A C}$ bisects $\overleftrightarrow{F E}$ at B .

7. In the diagram below of $\triangle A B C$, side $\overline{B C}$ is extended to point $D, \mathrm{~m} \angle A=x, \mathrm{~m} \angle B=2 x+15$, and $\mathrm{m} \angle A C D=5 x+5$. What is $\mathrm{m} \angle B$ ?

8. In the following diagram $k \| h$. Which of the following is $m \angle 1$ ?
(1) $12^{\circ}$
(3) $54^{\circ}$
(2) $126^{\circ}$
(4) $84^{\circ}$

9. Steve drew line segments $A B C D, E F G, B F$, and $C F$ as shown in the diagram below. Scalene $\triangle B F C$ is formed. Which statement will allow Steve to prove $\overline{A B C D} \| \overline{E F G}$ ?
1) $\angle C F G \cong \angle F C B$
2) $\angle A B F \cong \angle B F C$

3) $\angle E F B \cong \angle C F B$
4) $\angle C B F \cong \angle G F C$
10. In the diagram below of isosceles triangle $A B C, \overline{A B} \cong \overline{C B}$ and angle bisectors $\overline{A D}, \overline{B F}$, and $\overline{C E}$ are drawn and intersect at $X$. If $\mathrm{m} \angle B A C=50^{\circ}$, find $\mathrm{m} \angle A X C$.


Topic C- Rigid Motions

| Term | Definition | Diagram |
| :---: | :---: | :---: |
| Rigid Motions (Isometry) |  | ---------------------- -- - - - |
| Reflections |  |  |
| Rotations |  |  |
| Translation |  | $c \bigcup_{B}^{D} c \oint_{s}^{D}$ |

Example 1: Determine the line of reflection and label it $\ell$.

## Steps to finding the Line of Reflection:

1. Measure A to $A^{\prime}$ (or any corresponding pair of points)
2. Construct the perpendicular bisector of $A A^{\prime}$. This is the line of reflection.


Example 2: Reflect $\triangle A B C$ over line $m$. Steps to reflecting a figure over a line:

1) Point on $A$, make arc that will hit the line of reflection twice. (Label the intersections D and E)
2) Do NOT CHANGE THE SIZE OF THE COMPASS
3) Put sharp end on $D$, make an arc on the opposite side of line. Repeat for E .
4)The intersection point of these 2 circles opposite the line of reflection is now $\mathrm{A}^{\prime}$


Example 3: Reflect $\triangle A B C$ over the line $x=-1$ and state the coordinates of the image.


Example 4: Find the mark for the center of rotation for the transformation.

| Steps to finding the Center of Rotation |
| :--- |
| 1. Measure A to $A^{\prime}$ (or any <br> corresponding pair of points) |
| 2. Construct the perpendicular bisector <br> of $A A^{\prime}$. |
| 3. Repeat steps 1-2 for another pair of <br> corresponding points. |
| 4. The intersection of the perpendicular <br> bisectors is the center of rotation. |



Example 5: Construct a $60^{\circ}$ rotation of $\triangle A B C . R_{O, 60^{\circ}}(\triangle A B C)$

## Steps to rotate a figure given the center

 of rotation:1. Connect $O$ to $A$, then with compass (sharp end on O), create a circle centered at $O$ with radius $O A$. ( $\mathrm{A}^{\prime}$ will be on this circle $60^{\circ}$ counterclockwise from A)
2. Keep compass frozen move to $A$ and make an arc that intersects the first circle.
3. Label intersection on circle A'.


Example 6: The diagram below shows a clockwise rotation of $90^{\circ}$ degrees was performed on $\triangle J K L$ to create $\triangle X Y Z$. If $m \angle J=35^{\circ}$ and $m \angle Y=70^{\circ}$ find the measure of $\angle Z$. Explain your solution.


Example 7: If a regular octagon is rotated clockwise around its center, the minimum number of degrees it must be rotated to carry the octagon onto itself is


Example 8: Rotate $\triangle A B C 90^{\circ}$ about the origin and label the new $\triangle D E F$. State the coordinates of $\triangle D E F$ below.


Example 9: Translate $\triangle A B C$ along vector $\overrightarrow{W X}$ :


Steps to rotate a figure given a vector:
1.Measure compass to WX (vector)
2. Move pointer to A make arc (in direction vector shows)
3. Measure compass A to W
4. Pointer at $X$ make arc to hit $1^{\text {st }}$ arc
5. Label intersection point A' (Should look
 like A moved same distance and direction as $\overrightarrow{W X}$ )

Example 10: In the diagram below, $\triangle A B C$ has coordinates $A(-3,0), B(0,4)$ and $C(0,0)$. Graph, label, and state the coordinates of $\triangle A^{\prime \prime} B^{\prime \prime} C^{"}$ the image of $\triangle A B C$ after a reflection over the line $\mathrm{y}=0$ and a translation five units to the right and two units up.


## Mixed Practice with Topic C Pages 17-19

1. Which rigid motion has taken place?
a) $\qquad$
b) $\qquad$
c) $\qquad$
d) $\qquad$


2. Under which transformation would $\triangle A^{\prime} B^{\prime} C^{n}$, the image of $\triangle A B C$, not be congruent to $\triangle A B C$ ?
1) reflection over the $y$-axis
2) rotation of $90^{\circ}$ clockwise about the origin
3) translation of 3 units right and 2 units down
4) dilation with a scale factor of 2 centered at the origin
3. Which regular polygon has a minimum rotation of $45^{\circ}$ to carry the polygon onto itself?
(1) Octagon
(2) decagon
(3) hexagon
(4) pentagon
4. The graph below shows $\triangle A B C$ and its image, $\triangle A^{\prime \prime} B^{\prime \prime} C^{n}$. Describe a sequence of rigid motions which would map $\triangle A B C$ onto $\triangle A^{\prime \prime} B^{\prime \prime} C^{\prime \prime}$.

5. In the diagram below, congruent figures 1,2 , and 3 are drawn. Which sequence of transformations maps figure 1 onto figure 2 and then figure 2 onto figure 3 ?
1) a reflection followed by a translation
2) a rotation followed by a translation
3) a translation followed by a reflection
4) a translation followed by a rotation

6. Quadrilateral $A B C D$ is a parallelogram with diagonals $\overline{A C}$ and $\overline{B D}$ intersecting at $E$. Describe a single rigid motion that maps $\triangle A E D$ onto $\triangle C E B$.

7. In the diagram below, $\triangle A B C$ has coordinates $A(1,1), B(4,1)$, and $C(4,5)$. Graph, label, AND state the coordinates of $\triangle A$ " $B^{\prime \prime} C^{\prime \prime}$, the image of $\triangle A B C$ after the translation five units to the right and two units up followed by the reflection over the line $x=0$.

8. Describe a sequence of rigid motions which would map $A B C D$ onto $A^{\prime \prime} B^{\prime \prime} C^{\prime \prime} D^{\prime \prime}$.

9. The diagram below shows a rotation of $\theta$ degrees was performed on $\triangle A B C$ to create $A^{\prime} B^{\prime} C^{\prime}$. If $m \angle A=52^{\circ}$ and $m \angle C^{\prime}=40^{\circ}$ find the measure of $\angle B^{\prime}$. Explain your solution.

10. The image of $\triangle A B C$ after a rotation of $90 \circ$ clockwise about the origin is $\triangle D E F$. Which statement is not true?
1) $\overline{A B} \quad \overline{D E}$
2) $\overline{A C} \quad \overline{D F}$
3) $C \quad F$
4) $\overline{B C} \quad \overline{D E}$

11. In the diagram below, a square is graphed in the coordinate plane. A reflection over which line does not carry the square onto itself?
1) $y=x$
2) $y=2$
3) $x=5$
4) $x=2$
12. Find the line of reflection


13. Find the center of rotation

14. Describe a sequence of transformations that will map $\triangle A B C$ onto $\triangle D E F$ as shown below.


## Topic D - Congruency \& Proofs

| Diagram | Given | Conclusion Statement | Reason |
| :---: | :---: | :---: | :---: |
| $\stackrel{\square}{\square} \quad \stackrel{\circ}{\text { M }}$ | M is the midpoint of $\overline{A B}$ |  | A midpoint |
|  | $\overline{B D}$ is a median |  | A median |
|  | $\overline{A C}$ and $\overline{B D}$ bisect each other at E |  | A bisector |
|  | $\overline{B D}$ bisects $\angle A B C$ |  | A bisector |
|  | $\overline{B D} \perp \overline{A C}$ |  | Perpendicular lines |
|  | $\overline{P S}$ is an altitude |  | An altitude |
|  | $\overline{A B} \\| \overline{C D}$ |  | Given parallel lines cut by a transversal |
|  | The diagram |  |  |
|  | The Diagram |  |  |
|  | The Diagram |  |  |


|  |  |  |  |  |
| :--- | :--- | :--- | :--- | :--- |

## If two triangles are proven congruent we can then say

 "Corresponding ___or of congruent triangles areExample 1: Three of the four items listed can be used to establish congruence by ASA. Determine which one is NOT needed to prove $\triangle B C A \cong \triangle D C E$ by ASA ?
(1) C is the midpoint of $\overline{A E}$
(3) $\overline{A B} \cong \overline{E D}$
(2) $\angle \mathrm{B} \cong \angle \mathrm{D}$
(4) $\angle \mathrm{A} \cong \angle \mathrm{E}$


Example 2: Given $\triangle A C E$ and $\triangle A B F$ shown in the diagram to the right, with $\overline{A B} \cong \overline{A C}$. Which statement is needed to prove $A C E \quad A B F$ by $S A S \cong S A S$ ?


HINT: Separate $\triangle A C E$ and $\triangle A B F$

1) $\angle A C E \cong \angle A B F$
2) $\angle A E C \cong \angle A F B$
3) $\overline{B F} \cong \overline{C E}$
4) $\overline{A F} \cong \overline{A E}$

Example 3: To prove $\triangle A B C \cong \triangle D E F$ using Hypotenuse leg, what other information would you need?
(1) $\overline{A B} \cong \overline{D E}$
(3) $\overline{A C} \cong \overline{F D}$
(2) $\overline{A B} \cong \overline{F D}$
(4) $<A C B \cong<D F E$

Example 4: Determine congruence from the given information. (1) Create the congruence statement, and then (2) provide the congruence criteria (SSS, SAS, ASA, AAS, HL)

E is the midpoint of $\overline{A B}$ and $\overline{D C}$
(1) $\Delta \mathrm{EDA} \cong \Delta$ $\qquad$

(2) by $\qquad$


Example 5: Given: $\overline{M A} \| \overline{H T}$, and B is the midpoint of $\overline{H A}$. Prove: $\triangle M B A \cong \triangle T B H$


Precisely describe a rigid motion that would map $\triangle M B A$ onto $\triangle T B H$.

Example 7: Given: $\overline{B D} \perp \overline{A C}$ and $\overline{B D}$ bisects $\angle A B C$ Prove: $\overline{A D} \cong \overline{D C}$


Precisely describe a rigid motion that would map $\triangle A B D$ onto $\triangle C B D$.

## Mixed Practice with Topic D Pages 23-24

1. Tiffany notices that two congruent corresponding sides and the corresponding angle and says that these two triangles are congruent by SAS. Is she correct? Are the triangles congruent by SAS or some other congruence criteria? Explain your answer.

2. Given that $\overline{B D}$ bisects $\angle A D C$, fill in the conclusion statement and reason columns below based on the given.


| Conclusion Statement | Reason |
| :---: | :---: |
|  |  |
|  |  |

3. In the following diagram Joey is given that $\angle M B A \cong \angle A T M$ and $\angle T M A \cong \angle B A M$. Which sides does Joey know congruent in order for him to complete the proof of $\triangle M B A \cong \triangle A T M$ by AAS congruence criteria.
(1) $\overline{M H} \cong \overline{A H}$
(3) $\overline{M T} \cong \overline{A B}$
(2) $\angle B M A \cong \angle M A T$
(4) $\overline{M A} \cong \overline{M A}$

4. In the accompanying diagram, $\overline{C A} \perp \overline{A B}, \overline{E D} \perp \overline{D F}, \overline{E D} \| \overline{A B}, \overline{C E} \cong \overline{B F}, \overline{A B} \cong \overline{E D}$, and $\mathrm{m} \angle C A B=\mathrm{m} \angle F D E=90$. Which criteria could not be used to prove $\triangle A B C \cong \triangle D E F$ ?
(1) $\mathrm{SSS} \cong \mathrm{SSS}$
(2) $\mathrm{SAS} \cong S A S$
(3) AAS $\cong A A S$
(4) $\mathrm{HL} \cong \mathrm{HL}$


F
5. In $\triangle B A T$ and $\triangle C R E, \angle A \cong \angle R$ and $\overline{B A} \cong \overline{C R}$. Write one additional statement that could be used to prove that the two triangles are congruent. State the method that would be used to prove that the triangles are congruent
6. Given: $\overline{B D}$ bisects $\angle A B C, \overline{B D} \perp \overline{A C}$

Prove: $\overline{A B} \cong \overline{C B}$

7. Complete the partial proof below for the accompanying diagram by providing reasons for steps $3,6,8,9,10$ and 11 .

Given: $\overline{A F C D}, \overline{A B} \perp \overline{B C}, \overline{D E} \perp \overline{E F}, \overline{B C} \| \overline{F E}, \overline{A F} \cong \overline{C D}$
Prove: $\overline{A B} \cong \overline{D E}$


| Statements | Reasons |
| :--- | :--- |
| $1 \overline{A F C D}$ | 1 Given |
| $2 \overline{A B} \perp \overline{B C}, \overline{D E} \perp \overline{E F}$ | 2 Given |
| $3 \angle B$ and $\angle E$ are right angles. | 3 |
| $4 \angle B \cong \angle E$ | 4 All right angles are congruent. |
| $5 \overline{B C} \\| \overline{F E}$ | 5 Given |
| $6 \angle B C A \cong \angle E F D$ | 6 |
| $7 \overline{A F} \cong \overline{C D}$ | 7 Given |
| $8 \overline{F C} \cong \overline{F C}$ | 8 |
| $9 \overline{A C} \cong \overline{F D}$ | 9 |
| $10 \Delta A B C \cong \triangle D E F$ | 10 |
| $11 \overline{A B} \cong \overline{D E}$ | 11 |

8. Which statement is sufficient evidence that $\triangle D E F$ is congruent to $\triangle A B C$ ?
1) $A B=D E$ and $B C=E F$
2) $\angle D \cong \angle A, \angle B \cong \angle E, \angle C \cong \angle F$
3) There is a sequence of rigid motions that maps $\overline{A B}$ onto $\overline{D E}, \overline{B C}$ onto $\overline{E F}$, and $\overline{A C}$ onto $\overline{D F}$.
4) There is a sequence of rigid motions that maps point $A$ onto point $D, \overline{A B}$ onto $\overline{D E}$, and $\angle B$ onto $\angle E$.


## Topic E - Quadrilaterals \& Their Properties

|  | Parallelogram | Rectangle | Rhombus | Square | Trapezoid |
| :--- | :--- | :--- | :--- | :--- | :--- |
| At least one pair of opposite sides parallel |  |  |  |  |  |
| Opposite angles $\cong$ |  |  |  |  |  |
| Consecutive angles supplementary |  |  |  |  |  |
| Opposite sides $\cong$ |  |  |  |  |  |
| Opposite sides parallel |  |  |  |  |  |
| Diagonals bisect each other |  |  |  |  |  |
| Diagonals bisect angles |  |  |  |  |  |
| Diagonals $\perp$ to each other |  |  |  |  |  |
| Diagonals $\cong$ |  |  |  |  |  |
| Equiangular |  |  |  |  |  |
| Equilateral |  |  |  |  |  |

Example 1: In the diagram of parallelogram $A B C D$ shown below, $\overline{D C}$ is extended to $E$, and $\overline{B E}$ is drawn such that $\overline{B C} \cong \overline{C E}$. If $m \angle A=112^{\circ}$ what is $m \angle C B E$.


Example 2: The diagram below shows parallelogram $A B C D$ with diagonal $\overline{B D}, m \angle A=112^{\circ}$ and $m \angle B D C=32^{\circ}$. What is the measure of $A D B$ ?
(1) $32^{\circ}$
(2) $36^{\circ}$
(3) $144^{\circ}$
(4) $112^{\circ}$


Example 3: Quadrilateral MATH has diagonals $\overline{M T}$ and $\overline{A H}$. Which information is not sufficient to prove MATH is a parallelogram?

1) $\overline{M A} \cong \overline{T H}$ and $\overline{A T} \cong \overline{M H}$
2) $\overline{M A} \cong \overline{T H}$ and $\overline{M A} \| \overline{T H}$
3) $\overline{M T}$ and $\overline{A H}$ bisect each other.
4) $\overline{M A} \cong \overline{T H}$ and $\overline{A T} \| \overline{M H}$

Example 4: Which of the following group of quadrilaterals have congruent diagonals?

1) Rhombus, Square
2) Rhombus, Parallelogram, Square
3) Rectangle, Square
4) Rectangle, Rhombus, Square

Example 5: In parallelogram $Q R S T$ shown below, diagonal $\overline{T R}$ is drawn, $U$ and $V$ are points on $\overline{T S}$ and $\overline{Q R}$, respectively, and $\overline{U V}$ intersects $\overline{T R}$ at $W$. If $m \quad S=60^{\circ}, m \quad S R T=83^{\circ}$ and $m \quad T W U=35^{\circ}$, what is $m W V Q$ ?
(1) $37^{\circ}$
(2) $60^{\circ}$
(3) $72^{\circ}$
(4) $83^{\circ}$


Example 6: Given: In parallelogram $A B C D$ shown below, diagonals $\overline{A C}$ and $\overline{B D}$ intersect at $E$. Prove: $D E A$ BEC


Example 7: Quadrilateral $A B C D$, diagonal $\overline{A F E C}$ is shown in the diagram. Fill in the missing reasons below to complete the following proof.
Given: $\overline{A F} \cong \overline{C E}, \overline{B F} \perp \overline{A C}, \overline{D E} \perp \overline{A C}, \angle B A F \cong \angle D C E$ Prove: $A B C D$ is a parallelogram.


| STATEMENT | REASON |
| :--- | :--- |
| 1. $\overline{A F} \cong \overline{C E}$ | 1. Given |
| 2. $\overline{B F} \perp \overline{A C}, \overline{D E} \perp \overline{A C}$ | 2. Given |
| 3. $\angle B F A$ and $\angle D E C$ are right angles | 3. |
| 4. $\angle B F A \cong \angle D E C$ | 4. All right angles are congruent |
| 5. $\angle B A F \cong \angle D C E$ | 5. Given |
| 6. $\overline{B A} \\| \overline{D C}$ | 6. |
| 7. $\triangle B A F \cong \triangle D C E$ | 7. |
| 8. $\overline{B A} \cong \overline{D C}$ | 8. Corresponding sides of Congruent Triangles are Congruent |
| 9. $A B C D$ is a parallelogram. | 9. |

## Mixed Practice with Topic E Pages 27-28

1. In the diagram below, $A B C D$ is a parallelogram, $\overline{A B}$ is extended through $B$ to $E$ and $\overline{C E}$ is drawn. If $\overline{C E} \quad \overline{B E}$ and $m \quad D=112^{\circ}$, what is $m \quad E$ ? Explain your solution.

2. Given parallelogram $A B C D$ with diagonals $\overline{A C}$ and $\overline{B D}$ intersecting at $E$. Which statement must be true?
1) $\overline{B E} \cong \overline{C E}$
2) $\angle B A E \cong \angle D C E$
3) $\overline{A B} \cong \overline{B C}$
4) $\angle D A E \cong \angle C B E$
3. In the accompanying diagram of parallelogram $A B C D, m \angle B=5 x$, and $m \angle C=2 x+12$. Find the number of degrees in $\angle D$.

4. Parallelogram $A B C D$, with $m \quad C=85^{\circ}, m \quad C D F=52^{\circ}$ and $m \quad G F B=80^{\circ}$ find $m \quad F E A$.

5. Quadrilateral $A B C D$ with diagonals $\overline{A C}$ and $\overline{B D}$ is shown in the diagram below. Which information is not enough to prove $A B C D$ is a parallelogram?
(1) $\overline{A B} \cong \overline{C D}$ and $\overline{A B} \| \overline{C D}$
(2) $\overline{A B} \cong \overline{C D}$ and $\overline{B C} \cong \overline{A D}$
(3) $\overline{A B} \cong \overline{C D}$ and $\overline{B C} \| \overline{A D}$
(4) $\overline{A B} \| \overline{C D}$ and $\overline{B C} \| \overline{A D}$

6. In parallelogram $A B C D$, diagonals $\overline{A C}$ and $\overline{B D}$ intersect at $E$. Which statement does not prove parallelogram $A B C D$ is a rhombus?
1) $\overline{A C} \cong \overline{D B}$
2) $\overline{A B} \cong \overline{B C}$
3) $\overline{A C} \perp \overline{D B}$
4) $\overline{A C}$ bisects $\angle D C B$
7. Given: Rectangle $A B C D$, with N the midpoint of $\overline{C D}$ Prove: $\overline{B N} \cong \overline{A N}$

8. Given: In parallelogram $A B C D$ shown below, diagonals $\overline{A C}$ and $\overline{B D}$ intersect at $E$.

Prove: $\triangle A E B \cong \triangle C E D$

$\qquad$
$\qquad$

1. Which transformation would result in the perimeter of a triangle being different from the perimeter of its image?
1)Translation
2) Dilation
3) Reflection
4) Rotation
2. A parallelogram must be a rhombus when its
1) diagonals are perpendicular
2) opposite sides are parallel
3) diagonals are congruent
4) opposite sides are congruent
3. In the diagram below, congruent figures 1,2 , and 3 are drawn. Which sequence of transformations maps figure 1 onto figure 2 and then figure 2 onto figure 3 ?
1) a reflection followed by a translation
2) a rotation followed by a translation
3) a translation followed by a reflection
4) a translation followed by a rotation

4. Which regular polygon has a minimum rotation of $60^{\circ}$ to carry the polygon onto itself?
1) Octagon
2) decagon
3) hexagon
4) pentagon
5. Which construction is represented by these construction marks?
1) The angle bisector of $\angle A B C$
2) A perpendicular line $\overline{A C}$
3) The perpendicular bisector of $\overline{B C}$
4) Copying $\angle \mathrm{ABC}$

6. The diagram below shows the construction of the perpendicular bisector of $\overline{A B}$. Which statement is not true?
1) $A C=C B$
2) $C B=\frac{1}{2} A B$
3) $A C=2 A B$
4) $A C+C B=A B$

7. As shown in the diagram below $\overline{A B} \cong \overline{A E}$ and $\overline{A C} \cong \overline{A D}$. Which piece of information could be used to prove $\triangle A B D \cong \triangle A E C$ by SAS?
1) $\angle A B D \cong \angle A E C$
2) $\angle 1 \cong \angle 2$
3) $\angle A \cong \angle A$
4) $\angle A D B \cong \angle A C E$

8. Quadrilateral $A B C D$ is graphed on the set of axes below. When $A B C D$ is rotated $90^{\circ}$ in a counterclockwise direction about the origin, its image is quadrilateral $A^{\prime} B^{\prime} C^{\prime} D^{\prime}$. Is distance preserved under this rotation, and which coordinates are correct for the given vertex?
1) no and $C^{\prime}(1,2)$
2) no and $D^{\prime}(2,4)$
3) yes and $A^{\prime}(6,2)$
4) yes and $B^{\prime}(-3,4)$

9. Line segment $B D$ is the perpendicular bisector of $\overline{A C}$, and $\overline{A B} \quad \overline{C B}$. Which conclusion can not be proven?
1) $A \quad C$
2) $\overline{B D}$ is an altitude of triangle $A B C$.
3) Triangle $A B C$ is scalene.
4) $B D A=90^{\circ}$.

10. In the diagram of $\triangle A B C, \overline{B D}$ is drawn to side $\overline{A C}$. If $\mathrm{m} \angle A=35, \mathrm{~m} \angle A B D=25$, and $\mathrm{m} \angle C=60$, which type of triangle is $\triangle B C D$ ?
1) equilateral
2) scalene
3) obtuse
4) right

11. The diagram below shows a rotation of $\theta$ degrees was performed on $\triangle A B C$ to create $A^{\prime} B^{\prime} C^{\prime}$. If $m \angle A=52^{\circ}$ and $m \angle C^{\prime}=40^{\circ}$ find the measure of $\angle B^{\prime}$. Explain your solution.

12. Given: In parallelogram $A B C D$ shown below, diagonals $\overline{A C}$ and $\overline{B D}$ intersect at $E$. Prove: $D E A$ BEC

13. Triangle $A B C$ is graphed on the set of axes below. Graph, label and state the coordinates of $\triangle A^{\prime \prime} B^{\prime \prime} C^{\prime \prime}$, the image of $\triangle A B C$ after a reflection over the line $x=1$ and a translation right 1 down 3.

14. In the diagram below, $A B C D$ is a parallelogram, $\overline{A B}$ is extended through $B$ to $E$, and $\overline{C E}$ is drawn. If $\overline{C E} \cong \overline{B E}$ and $\mathrm{m} \angle D=112^{\circ}$, what is $\mathrm{m} \angle E$ ?

15. Construct the line of reflection $\ell$ for the transformation shown.

16. Determine congruence from the given information. (1) Create the congruence statement, and then (2) provide the congruence criteria (SSS, SAS, ASA, AAS, HL)

$$
\overline{A C} \text { bisects } \angle B A D \text { and } \overline{A C} \perp \overline{B D}
$$

(1) $\triangle \mathrm{ACB} \cong \Delta$ $\qquad$ (2)by $\qquad$

17. Quadrilateral $A B C D$ is a parallelogram with diagonals $\overline{A C}$ and $\overline{B D}$ intersecting at $E$. Describe a single rigid motion that maps $\triangle A E D$ onto $\triangle C E B$.


