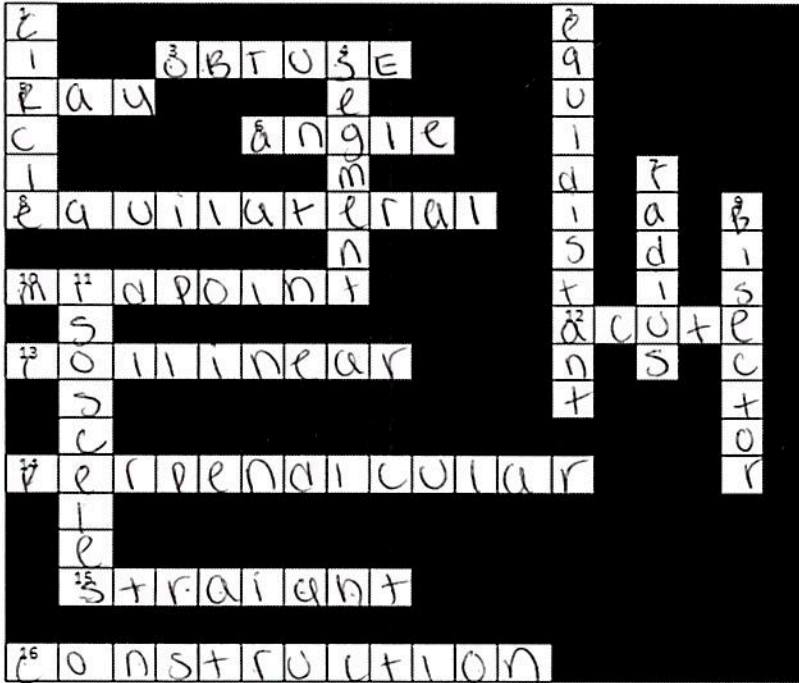


MODULE 1 REVIEW

TOPIC	Concepts Include	CC Standard	Page #
Topic A: Basic Constructions	<ul style="list-style-type: none"> • Construct an Equilateral Triangle, • Copy and Bisect an Angle, • Construct a Perpendicular Bisector, • Points of Concurrencies 	G-CO.1, G-CO.12, G-CO.13	2 - 8
Topic B: Unknown Angles	<ul style="list-style-type: none"> • Solving for Unknown angles, • Parallel lines and transversals, • Exterior angle theorem, • Auxiliary lines, • proving known facts 	G. CO-9	9 - 13
Topic C: Transformations/ Rigid Motions	<ul style="list-style-type: none"> • Rotations, reflections, translations, • Symmetry, • Sequence of rigid motions, • Transformations on the coordinate plane. 	G-CO.2-G-CO.7 G-CO.12	14 - 19
Topic D: Congruence	<ul style="list-style-type: none"> • Congruence Criteria—SAS, ASA, SSS, SAA and HL, • Proving triangles congruent, • Isosceles triangles, • Congruence in terms of rigid motions, • Corresponding parts of \cong triangles. 	G-CO.7, G-CO.8	20 - 24
Topic E: Proving Properties of Geometric Figures	<ul style="list-style-type: none"> • Properties of parallelograms, • Parallelogram proofs, • Mid-segment of a triangle, • Centroid of a triangle. 	G-CO.9, G-CO.10, G-CO.11	25 - 28
	MIXED REVIEW		29-32

Topic A- Vocabulary and Constructions


Example 1: Fill in the puzzle below using the vocabulary listed in the word bank.



Word Bank:

Collinear	Angle	Bisector
Obtuse	Ray	Isosceles
Midpoint	Acute	Segment
Perpendicular	Straight	Radius
Construction	Circle	Equidistant
Equilateral		

ACROSS

3. An angle measuring more than 90 and less than 180 degrees obtuse
5. A part of a line starting at one endpoint and going on forever through the other point on the line ray
6. Two noncollinear rays with a common endpoint form an angle 
8. A triangle with all sides and all angles congruent equilateral
10. A point that divides a line segment into two congruent halves midpoint
12. An angle less than 90 degrees acute
13. Points that lie on the same line collinear
14. Lines that form a right angle perpendicular
15. An angle measuring 180 degrees straight
16. A set of instructions for drawing points, lines, circles and figures in a plane construction

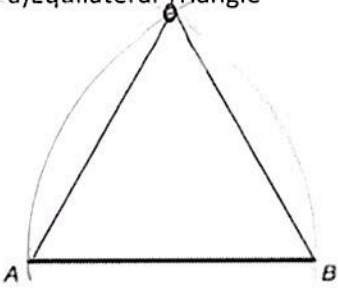
DOWN

1. A figure with a center point and all points the same distance away from the center circle
2. Point B is said to be equidistant from A and C if $AB=BC$
4. A part of a line between two endpoints segment
7. The distance from the center of the circle to any point on the circumference radius
9. A ray that divides an angle into two congruent parts bisector
11. A triangle with two equal legs and two equal base angles isosceles

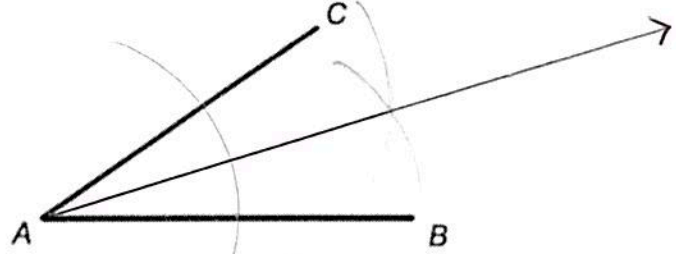
Constructions

Example 2: Using a straightedge and a compass, construct the following:

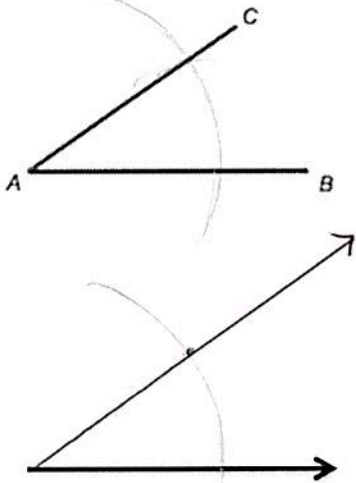
a) Equilateral Triangle



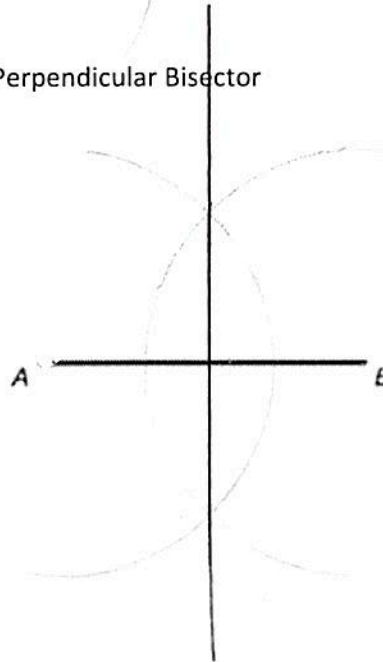
b) Angle Bisector



c) Copy an Angle



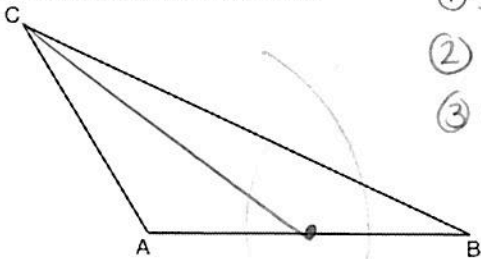
d) Perpendicular Bisector



Term	Definition
Median	A segment drawn from one vertex of a triangle to the _____ of the opposite side.
Altitude	A segment drawn from one vertex of a triangle _____ to the opposite side.

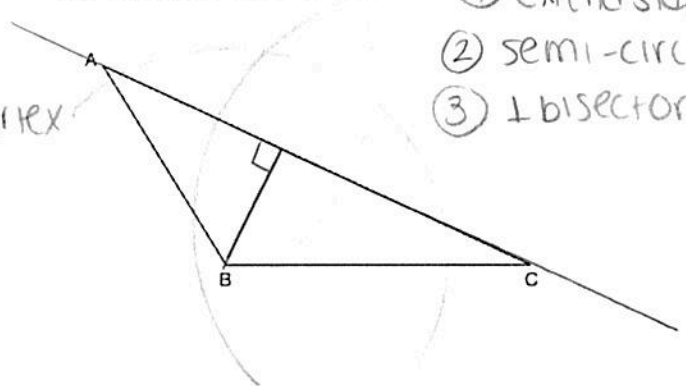
Example 3: In the diagram of $\triangle ABC$ shown below, use a compass and straightedge to construct the following:

a) Median from C to \overline{AB} .



- ① \perp bisector
- ② midpoint
- ③ connect to opp. vertex

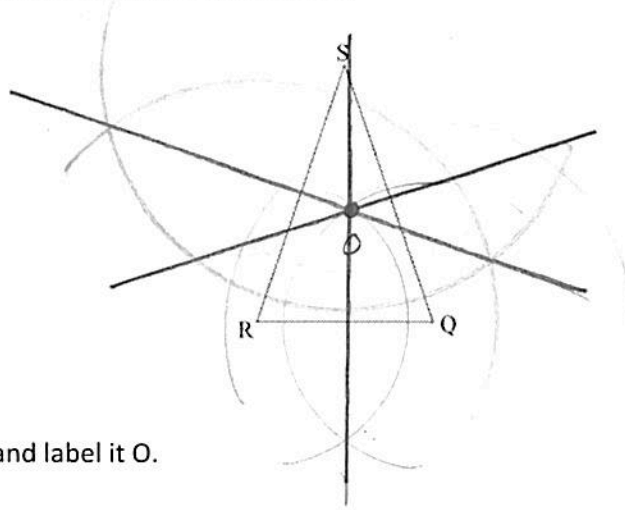
b) Altitude from B to \overline{AC}



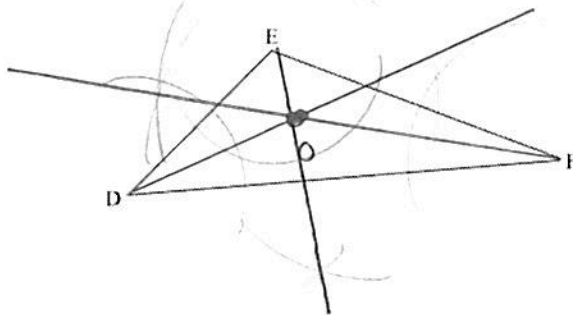
- ① extend side
- ② semi-circle
- ③ \perp bisector

Term	Definition
Circumcenter	Point of concurrency of 3 <u>perpendicular bisectors</u> in a triangle.
Incenter	Point of concurrency of 3 <u>angle bisectors</u> in a triangle.

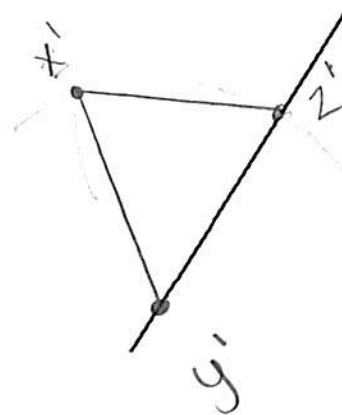
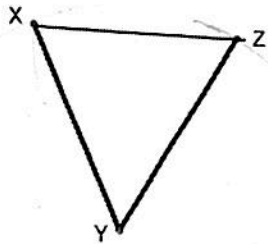
Example 4: a) Construct the circumcenter of $\triangle RSQ$ and label it O.



b) Construct the incenter of $\triangle DEF$ and label it O.



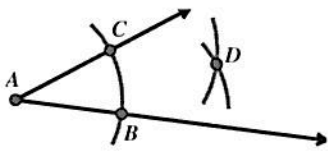
Example 5: Using a compass and a straightedge, on the line shown, construct $\triangle X'Y'Z'$, such that $\triangle X'Y'Z' \cong \triangle XYZ$. [Leave all construction marks]



Mixed Practice with Topic A Pages 5-8

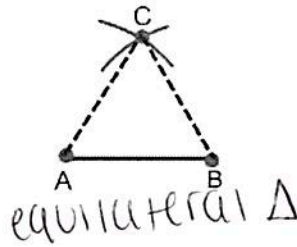
1. Identify the construction we have covered matches each diagram.

Diagram 1



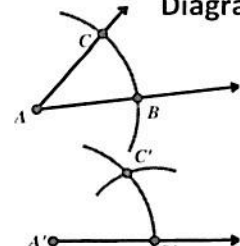
angle bisector

Diagram 2



equilateral Δ

Diagram 3

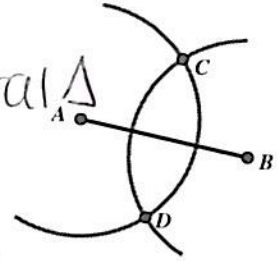


copy an \angle

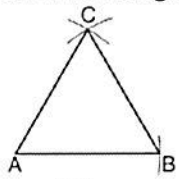
2. A student has done the following construction. What was this student attempting to construct? Is there more than one thing that the student could be constructing? Explain.

a \perp bisector

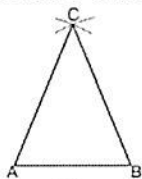
He also can be constructing an equilateral Δ or a square, or a midpoint or right \angle 's



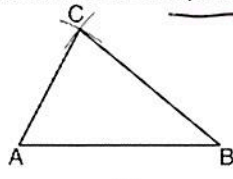
3. Which diagram represents a correct construction of equilateral ΔABC , given side \overline{AB} ?



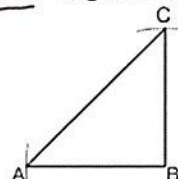
(1)



(2)



(3)



(4)

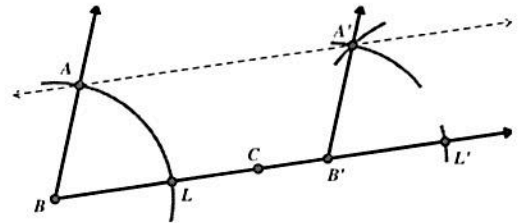
4. Which construction is completed in the diagram below to create parallel lines?

1) The angle bisector of $\angle ABL$

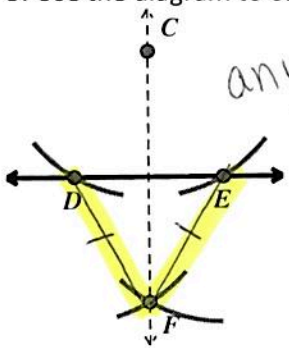
2) The perpendicular bisector of \overline{BC}

3) A perpendicular line \overline{BA}

4) Copying $\angle ABL$

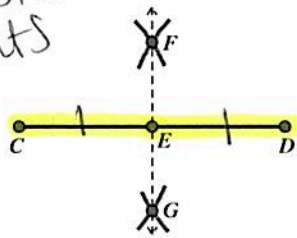


5. Use the diagram to complete the relationship. (The compass was constant for each individual construction.)

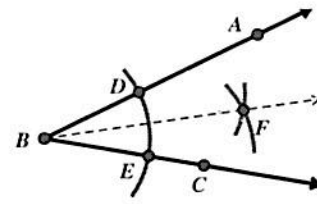


a) $DF = EF$

any point on \perp bisector is equidistant from endpoints



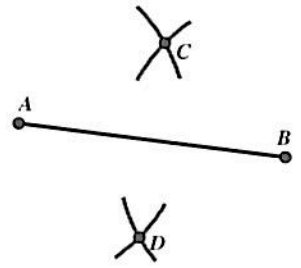
b) $CE = ED$



c) $m\angle ABF = \angle CBF$

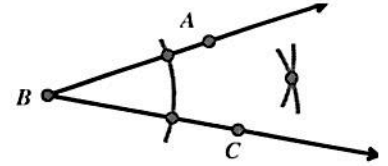
6. A teacher finds a paper on the ground in the classroom. When she looks at it carefully she realizes it is from her geometry class because it has a construction on it. Which of the following constructions is **NOT FOUND** directly from this student's work?

- 1) The midpoint of \overline{AB} ✓ 3) The perpendicular bisector of \overline{AB} ✓
 2) A perpendicular line to \overline{AB} ✓ (4) The angle bisector of $\angle CAB$



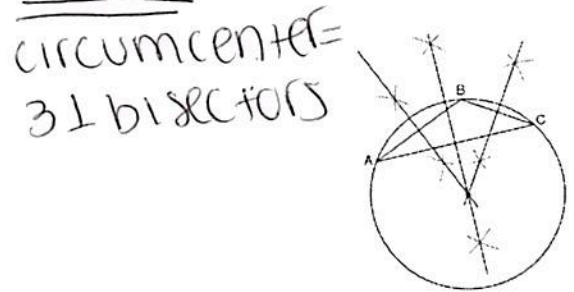
7. Which construction is represented by these construction marks?

- 1) Copying $\angle ABC$ 3) The perpendicular bisector of \overline{BC}
 (2) The angle bisector of $\angle ABC$ 4) A perpendicular line \overline{AC}



8. The diagram below shows the construction of the center of the circle circumscribed about $\triangle ABC$. This construction represents how to find the intersection of

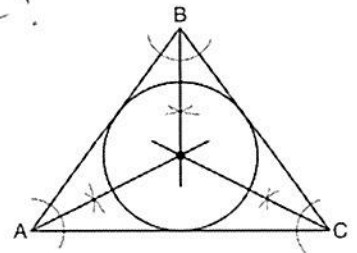
- 1) the angle bisectors of $\triangle ABC$
 2) the medians to the sides of $\triangle ABC$
 3) the altitudes to the sides of $\triangle ABC$
 (4) the perpendicular bisectors of the sides of $\triangle ABC$



9. Which geometric principle is used in the construction shown below?

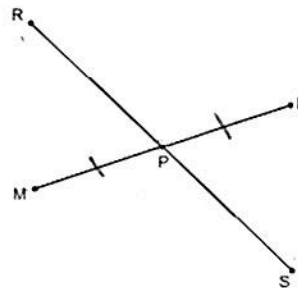
- (1) The intersection of the angle bisectors of a triangle is the center of the inscribed circle
 2) The intersection of the angle bisectors of a triangle is the center of the circumscribed circle
 3) The intersection of the perpendicular bisectors of the sides of a triangle is the center of the inscribed circle.
 4) The intersection of the perpendicular bisectors of the sides of a triangle is the center of the circumscribed circle

INCENTER!

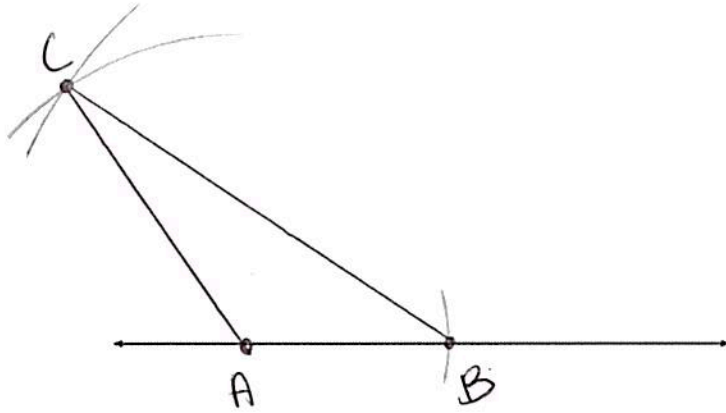
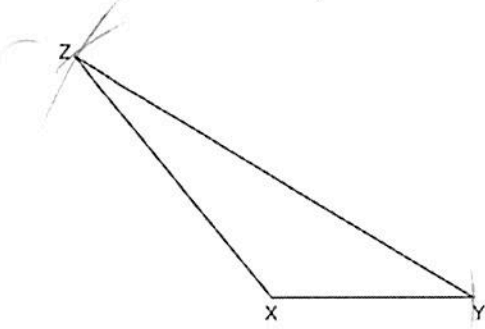


10. In the diagram below, it is given that \overline{RS} bisects \overline{MN} at point P. Which of the following statements does **not** have to be true?

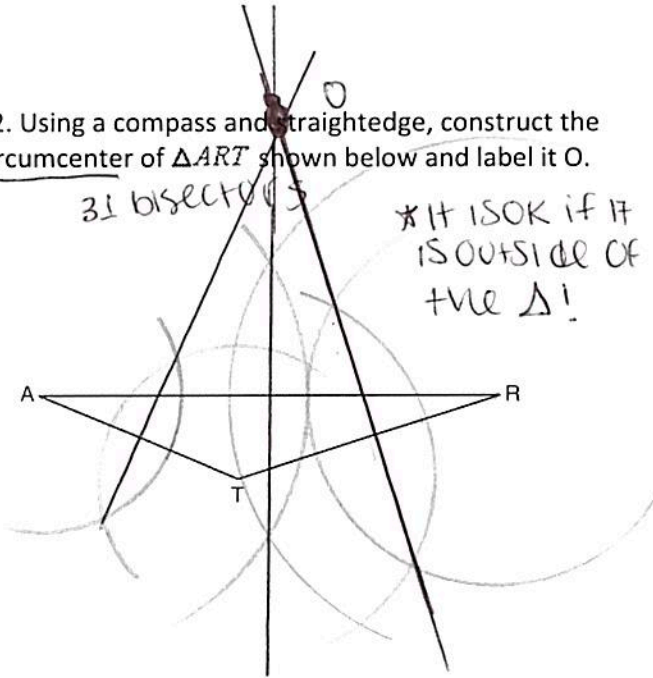
- 1) $MP = NP$ ✓
 2) $RP + PS = RS$ ✓
 (3) P is the midpoint of \overline{RS}
 4) P is the midpoint of \overline{MN} ✓



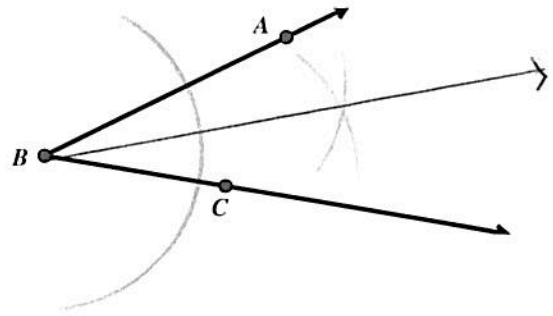
11. Triangle XYZ is shown below. Using a compass and straightedge, on the line below, construct and label $\triangle ABC$, such that $\triangle ABC \cong \triangle XYZ$. [Leave all construction marks.]



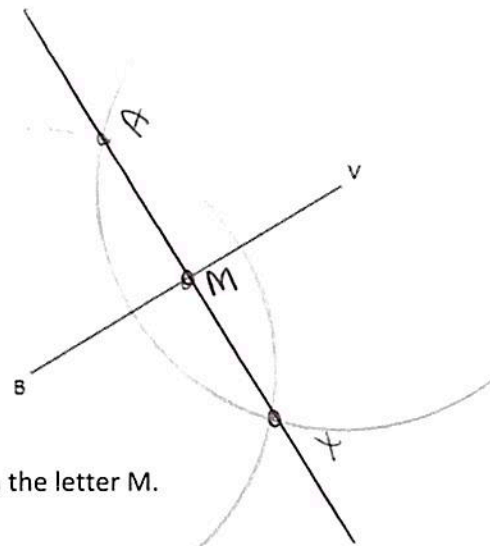
12. Using a compass and straightedge, construct the circumcenter of $\triangle ART$ shown below and label it O.



13. Construct the angle bisector of $\angle ABC$.



14. Using a compass and a straightedge, construct the perpendicular bisector of the \overline{BV} . Label it \overline{AX} .



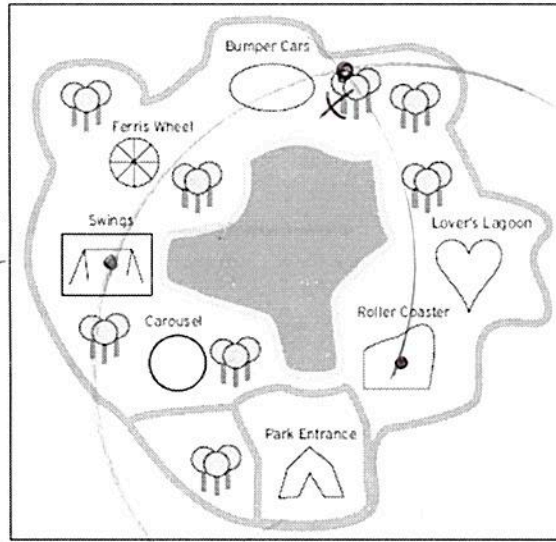
b) Identify the midpoint of the line segment with the letter M.

c) What can you say about any point on \overline{AX} in relation to endpoints B and V.

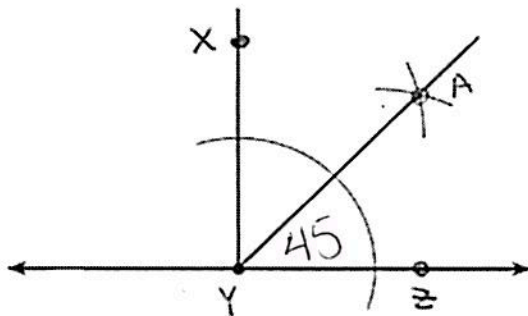
any point on \overline{AX} is equidistant from B & V

15. Jo wants to open a new soft drink stand at an amusement park. She looks at the map of the amusement park and notes the 2 most popular rides: roller coaster, and swings. Jo decides to locate the stand so that it is the same distance from these 2 rides. Why do you think that Jo wants the stand located at a point equidistant from the 2 most popular rides? Indicate where the stand should be placed with the letter X.

she would place it between the 2 most popular rides so people must pass the stand on their way from one to the next

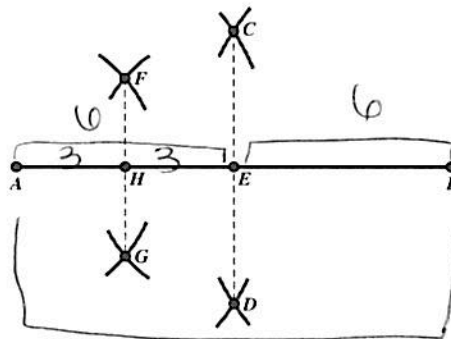


16. Right angle, $\angle XYZ$, was bisected below. What is the measure of $\angle AYZ$? **Explain** how you came to your solution.



AN \angle bisector creates 2 \cong \angle 's
 $90 \div 2 = 45^\circ$

17. If the length of \overline{AB} is 12 cm, what is the length of \overline{AH} after the construction was performed below?



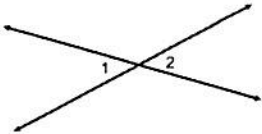

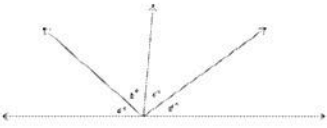
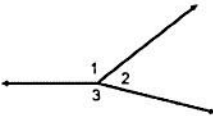
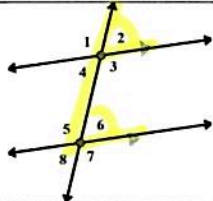
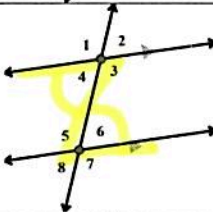
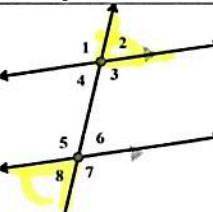
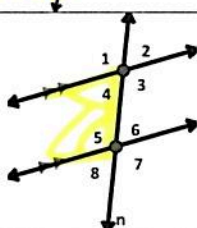
$$|AH| = 3$$

12

\perp bisectors!

Topic B- Important Geometry Facts and Theorems

Fill in the "Fact/Discovery" column based on geometry facts you have learned!

Types of Angles	Diagram	Fact/Discovery
Vertical Angles		vertical \angle 's are \cong
Complementary & Supplementary Angles		complementary = 90° supplementary = 180°
Adjacent Angles on a Line	 <p style="text-align: center;">$a + b + c + d = 180$</p>	\angle 's on a line sum to 180°
Angles around a Point		\angle 's at a point sum to 360°
Corresponding Angles		corresponding \angle 's are \cong
Alternate Interior Angles		alt. int. \angle 's are \cong
Alternate Exterior Angles		alt. ext. \angle 's are \cong
Same Side Interior Angles		same side int. \angle 's are supplementary

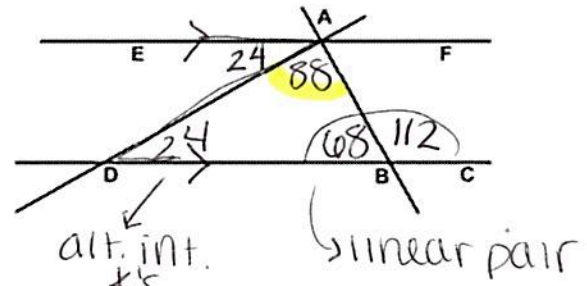
Fill in the "Fact/Discovery" column based on geometry facts you have learned!

	Diagram	Fact/Discovery
\angle Sum of Δ		\angle 's in a Δ sum to 180°
Isosceles Δ		Isosceles Δ has 2 \cong sides and 2 \cong base \angle 's
Exterior \angle of a Δ Theorem		The exterior \angle = to the sum of the 2 non-adjacent interior \angle 's (remote \angle 's)

Example 1: In the diagram below, $EF \parallel DC$, $m\angle ABC = 112^\circ$ and $m\angle EAD = 24^\circ$. What is the measure of $\angle DAB$?

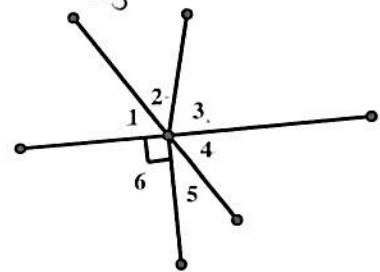
$$180 - (24 + 68) = 88^\circ = \angle DAB$$

\angle 's in a Δ sum to 180°



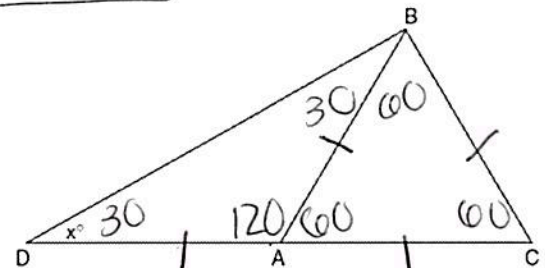
Example 2: Use the diagram to determine the answers.

- a) $\angle 2$ and $\angle 4$ are vertical angles. T or (F)
- b) $\angle 1$, $\angle 2$, and $\angle 3$ add to 180° . (T) or F
- c) $\angle 1$ and $\angle 6$ are complementary. T or (F)
- d) $\angle 3$ and $\angle 4$ are adjacent angles. (T) or F



Example 3: In the accompanying diagram of ΔBCD , ΔABC is an equilateral triangle and $AD = AB$. What is the value of x , in degrees? **Explain** how you reached your solution.

\angle 's of an equilateral $\Delta = 60^\circ$;
 \angle 's on a line sum to 180
 and base \angle 's of an isosceles Δ are \cong



$$\frac{180 - 120}{2} = 30$$

$$\boxed{x = 30^\circ}$$

Example 4: In the diagram of $\triangle ABC$ below, \overline{AB} is extended to point D . If $m\angle CAB = x + 40$, $m\angle ACB = 3x + 10$, $m\angle CBD = 6x$, what is $m\angle CAB$?

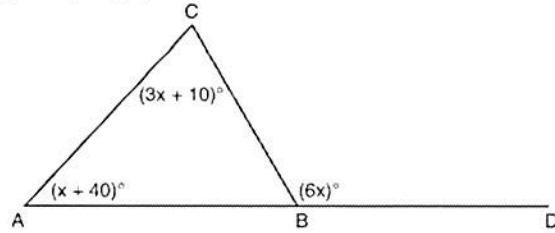
exterior angle theorem!

$$6x = x + 40 + 3x + 10$$

$$6x = 4x + 50$$

$$\begin{array}{r} 6x \\ -4x \\ \hline 2x = 50 \\ x = 25 \end{array}$$

$$m\angle CAB = 25 + 40 = \boxed{65^\circ}$$



Example 5: Fill in the missing reasons for steps 2 and 3 to prove the sum of the angles of a triangle is 180° .

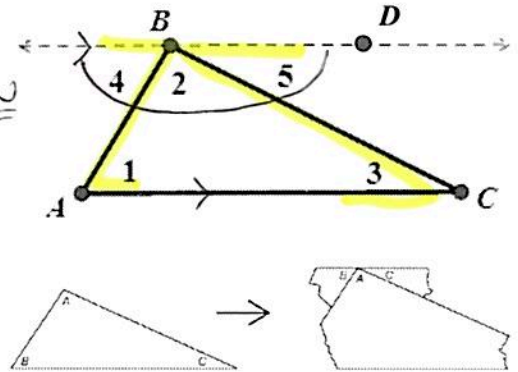
Given: $\triangle ABC$

Prove: $m\angle 1 + m\angle 2 + m\angle 3 = 180^\circ$

Construct an auxiliary line parallel to \overline{AC} through B

STATEMENT	REASON
1. $\overline{AC} \parallel \overline{BD}$	1. Given (Auxiliary Line)
2. $m\angle 4 = m\angle 1$ $m\angle 5 = m\angle 3$	2. alt. int. \angle 's are \cong
3. $m\angle 4 + m\angle 2 + m\angle 5 = 180^\circ$	3. \angle 's on a line sum to 180°
4. $m\angle 1 + m\angle 2 + m\angle 3 = 180^\circ$	4. Substitution Property

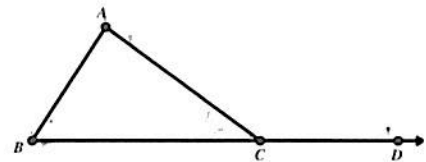
a line you create in diagram to prove known facts



Example 6: Fill in the missing reasons for steps 2 and 3 to prove that the exterior angle is equal to the sum of the two remote angles.

Given: $\triangle ABC$ with external angle, $\angle ACD$.

Prove: $m\angle ACD = m\angle B + m\angle A$

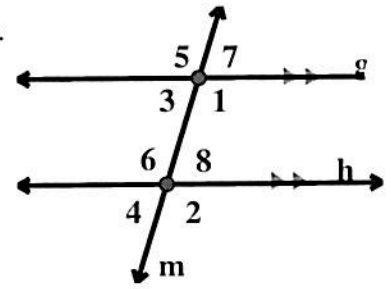


STATEMENT	REASON
1) $\triangle ABC$ with external angle, $\angle ACD$.	1) Given
2) $m\angle ACB + m\angle B + m\angle A = 180^\circ$	2) \angle 's in a \triangle sum to 180
3) $m\angle ACB + m\angle ACD = 180^\circ$	3) \angle 's on a line sum to 180
4) $m\angle ACB + m\angle ACD = m\angle ACB + m\angle B + m\angle A$	4) Substitution Property (both = 180° so must = each other)
5) $m\angle ACD = m\angle B + m\angle A$	5) Subtraction Property (subtracted $m\angle ACB$ both sides)

Mixed Practice with Topic B Pages 12-13

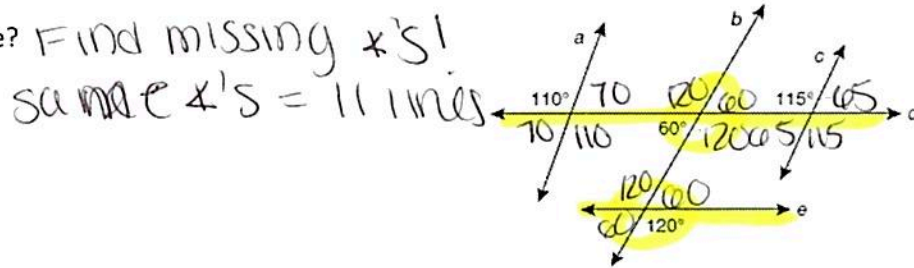
1. In the following diagram $g \parallel h$. State an angle that is congruent to $\angle 7$ and **explain** why.

$\angle 7 \cong \angle 8$ and $\angle 4$
 \downarrow \downarrow
 corresp. \angle 's are \cong
 alt. ext. \angle 's are \cong



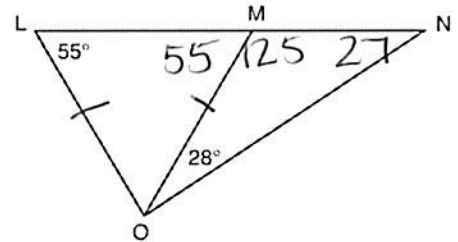
2. Based on the diagram, which statement is true? Find missing \angle 's!

- 1) $a \parallel b$
- 2) $a \parallel c$
- 3) $b \parallel c$
- 4) $d \parallel e$



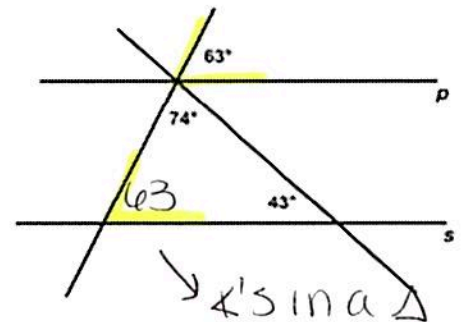
3. In the diagram below, $\triangle LMO$ is isosceles with $LO = MO$. If $m\angle L = 55$ and $m\angle NOM = 28$, what is $m\angle N$? **Explain** your solution.

isosceles \triangle 's have \cong base \angle 's
 \angle 's on a line sum to 180
 \angle 's in a \triangle sum to 180 so $\angle N = 27^\circ$



4. In the diagram below, lines p and s are cut by transversals. The angles are marked as shown. Explain why p and s must be parallel.

$p \parallel s$ b/c corresponding \angle 's are \cong



5. If the measures of the angles of a triangle are represented by $2x$, $3x - 15$, and $7x + 15$, the triangle is

- 1) an isosceles triangle
- 2) a right triangle
- 3) an acute triangle
- 4) an equiangular triangle

$$2x + 3x - 15 + 7x + 15 = 180$$

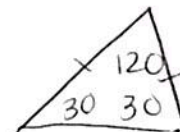
$$12x = 180$$

$$x = 15$$

$$2(15) = 30$$

$$3(15) - 15 = 30$$

$$7(15) + 15 = 120$$



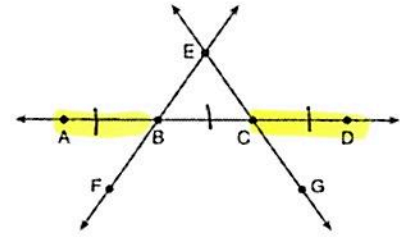
6. In the diagram below, \overline{FE} bisects \overline{AC} at B, and \overline{GE} bisects \overline{BD} at C. Which statement is always true?

(1) $\overline{AB} \cong \overline{DC}$

(3) \overline{BD} bisects \overline{GE} at C.

(2) $\overline{FB} \cong \overline{EB}$

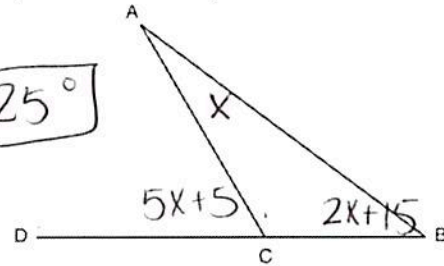
(4) \overline{AC} bisects \overline{FE} at B.



7. In the diagram below of $\triangle ABC$, side \overline{BC} is extended to point D, $m\angle A = x$, $m\angle B = 2x + 15$, and $m\angle ACD = 5x + 5$. What is $m\angle B$? exterior \angle theorem

$$\begin{aligned} x + 2x + 15 &= 5x + 5 \\ 3x + 15 &= 5x + 5 \\ -3x - 5 &\quad -3x - 5 \\ \hline 10 &= 2x \\ x &= 5 \end{aligned}$$

$$m\angle B = 2(5) + 15 = 25^\circ$$



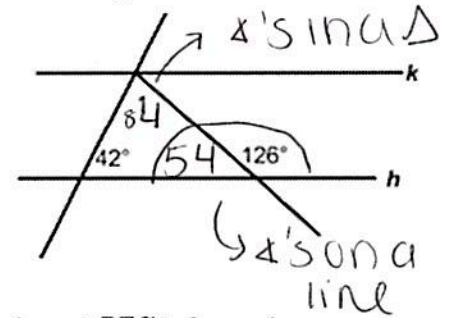
8. In the following diagram $k \parallel h$. Which of the following is $m\angle 1$?

(1) 12°

(3) 54°

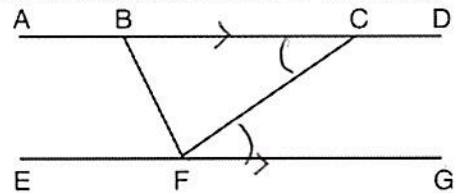
(2) 126°

(4) 84°

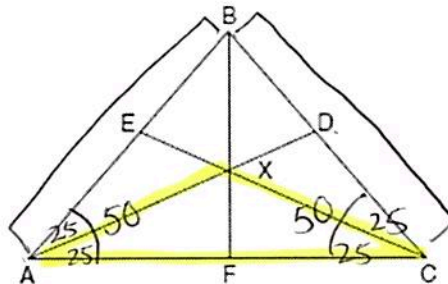


9. Steve drew line segments $ABCD$, EFG , BF , and CF as shown in the diagram below. Scalene $\triangle BFC$ is formed. Which statement will allow Steve to prove $\overline{ABCD} \parallel \overline{EFG}$?

- (1) $\angle CFG \cong \angle FCB$ alt. int. \angle 's are $\underline{\underline{N}}$
 (2) $\angle ABF \cong \angle BFC$
 (3) $\angle EFB \cong \angle CFB$
 (4) $\angle CBF \cong \angle GFC$



10. In the diagram below of isosceles triangle ABC , $\overline{AB} \cong \overline{CB}$ and angle bisectors \overline{AD} , \overline{BF} , and \overline{CE} are drawn and intersect at X. If $m\angle BAC = 50^\circ$, find $m\angle AXC$.

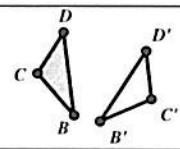
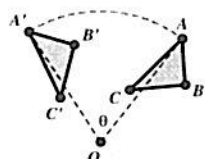
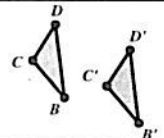


$$180 - (25 + 25) = 130$$

$$m\angle AXC = 130^\circ$$

\hookrightarrow isosceles \triangle !

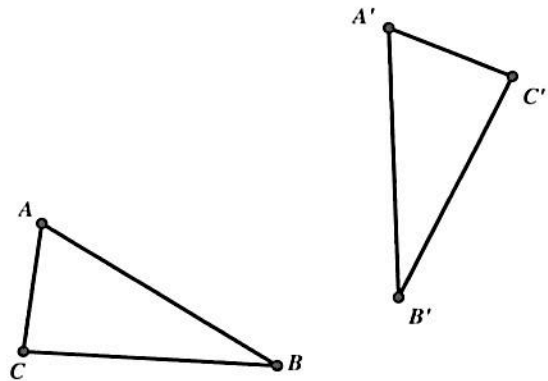
Topic C- Rigid Motions

Term	Definition	Diagram
Rigid Motions (Isometry)		-----
Reflections		
Rotations		
Translation		

Example 1: Determine the line of reflection and label it ℓ .

Steps to finding the Line of Reflection:

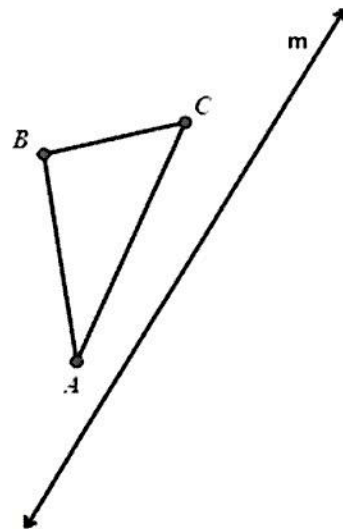
1. Measure A to A' (or any corresponding pair of points)
2. Construct the perpendicular bisector of AA'. This is the line of reflection.



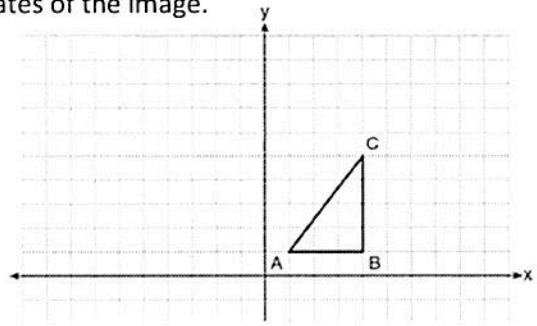
Example 2: Reflect $\triangle ABC$ over line m .

Steps to reflecting a figure over a line:

- 1) Point on A, make arc that will hit the line of reflection twice. (Label the intersections D and E)
- 2) Do NOT CHANGE THE SIZE OF THE COMPASS
- 3) Put sharp end on D, make an arc on the opposite side of line. Repeat for E.
- 4) The intersection point of these 2 circles opposite the line of reflection is now A'

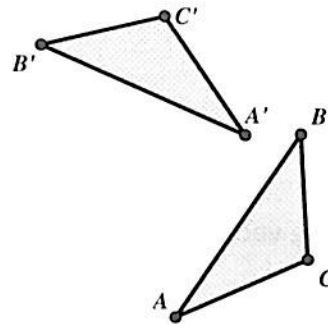


Example 3: Reflect $\triangle ABC$ over the line $x = -1$ and state the coordinates of the image.



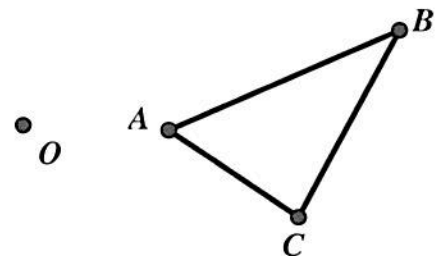
Example 4: Find the mark for the center of rotation for the transformation.

Steps to finding the Center of Rotation
1. Measure A to A' (or any corresponding pair of points)
2. Construct the perpendicular bisector of AA'.
3. Repeat steps 1-2 for another pair of corresponding points.
4. The intersection of the perpendicular bisectors is the center of rotation.

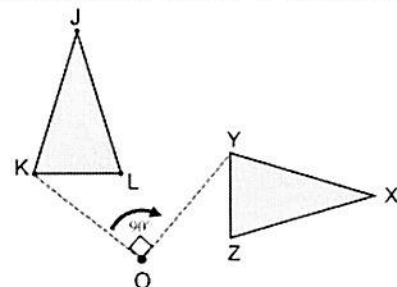


Example 5: Construct a 60° rotation of $\triangle ABC$. $R_{O,60^\circ}(\triangle ABC)$

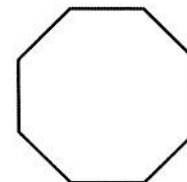
Steps to rotate a figure given the center of rotation:
1. Connect O to A, then with compass (sharp end on O), create a circle centered at O with radius OA. (A' will be on this circle 60° counterclockwise from A)
2. Keep compass frozen move to A and make an arc that intersects the first circle.
3. Label intersection on circle A'.



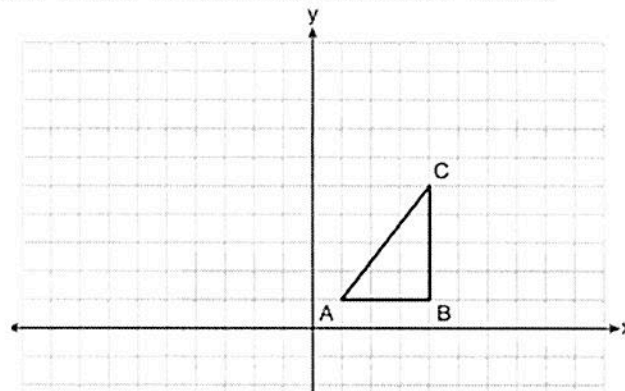
Example 6: The diagram below shows a clockwise rotation of 90° degrees was performed on $\triangle JKL$ to create $\triangle XYZ$. If $m\angle J = 35^\circ$ and $m\angle Y = 70^\circ$ find the measure of $\angle Z$. **Explain** your solution.



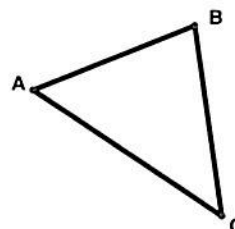
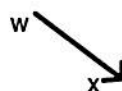
Example 7: If a regular octagon is rotated clockwise around its center, the minimum number of degrees it must be rotated to carry the octagon onto itself is



Example 8: Rotate $\triangle ABC$ 90° about the origin and label the new $\triangle DEF$. State the coordinates of $\triangle DEF$ below.

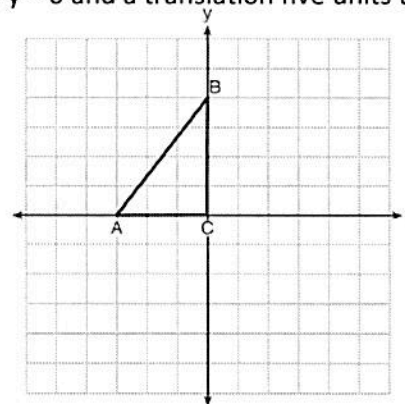


Example 9: Translate $\triangle ABC$ along vector \overrightarrow{WX} :



Steps to rotate a figure given a vector:
1. Measure compass to WX (vector)
2. Move pointer to A make arc (in direction vector shows)
3. Measure compass A to W
4. Pointer at X make arc to hit 1 st arc
5. Label intersection point A' (Should look like A moved same distance and direction as \overrightarrow{WX})

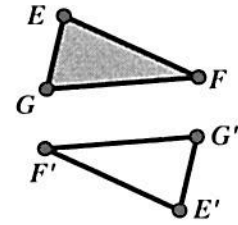
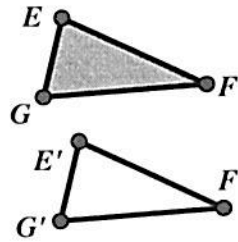
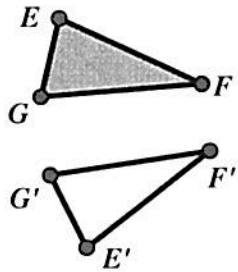
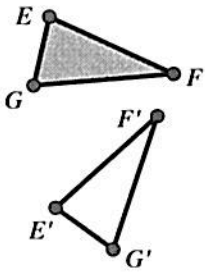
Example 10: In the diagram below, $\triangle ABC$ has coordinates $A(-3, 0)$, $B(0, 4)$ and $C(0, 0)$. Graph, label, and state the coordinates of $\triangle A''B''C''$ the image of $\triangle ABC$ after a reflection over the line $y = 0$ and a translation five units to the right and two units up.



Mixed Practice with Topic C Pages 17-19

1. Which rigid motion has taken place?

a) _____ b) _____ c) _____ d) _____



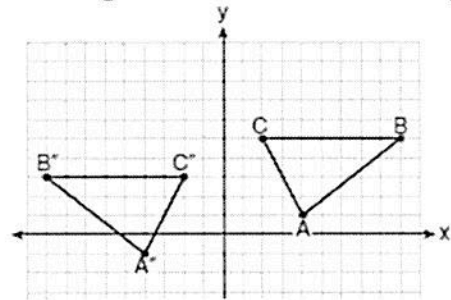
2. Under which transformation would $\triangle A'B'C'$, the image of $\triangle ABC$, *not* be congruent to $\triangle ABC$?

- 1) reflection over the y -axis
- 2) rotation of 90° clockwise about the origin
- 3) translation of 3 units right and 2 units down
- 4) dilation with a scale factor of 2 centered at the origin

3. Which regular polygon has a minimum rotation of 45° to carry the polygon onto itself?

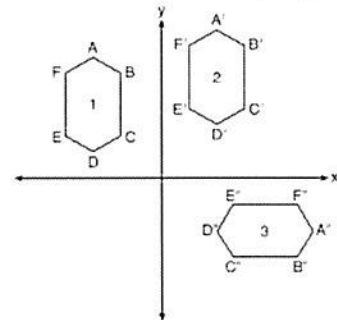
- (1) Octagon (2) decagon (3) hexagon (4) pentagon

4. The graph below shows $\triangle ABC$ and its image, $\triangle A''B''C''$. Describe a sequence of rigid motions which would map $\triangle ABC$ onto $\triangle A''B''C''$.

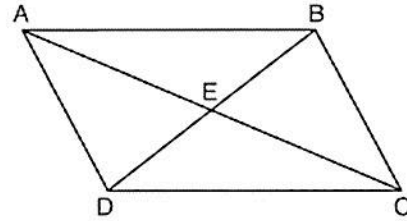


5. In the diagram below, congruent figures 1, 2, and 3 are drawn. Which sequence of transformations maps figure 1 onto figure 2 and then figure 2 onto figure 3?

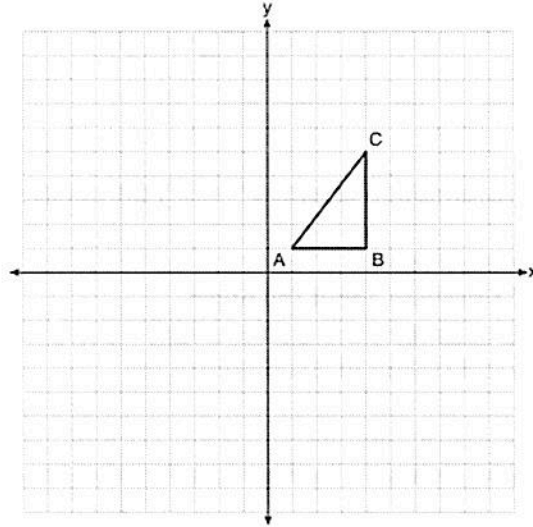
- 1) a reflection followed by a translation
- 2) a rotation followed by a translation
- 3) a translation followed by a reflection
- 4) a translation followed by a rotation



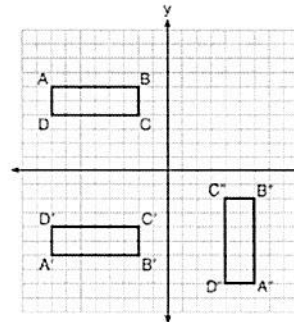
6. Quadrilateral $ABCD$ is a parallelogram with diagonals \overline{AC} and \overline{BD} intersecting at E . Describe a single rigid motion that maps $\triangle AED$ onto $\triangle CEB$.



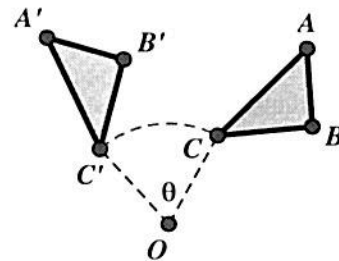
7. In the diagram below, $\triangle ABC$ has coordinates $A(1,1)$, $B(4,1)$, and $C(4,5)$. Graph, label, AND state the coordinates of $\triangle A''B''C''$, the image of $\triangle ABC$ after the translation five units to the right and two units up followed by the reflection over the line $x=0$.



8. Describe a sequence of rigid motions which would map $ABCD$ onto $A''B''C''D''$.

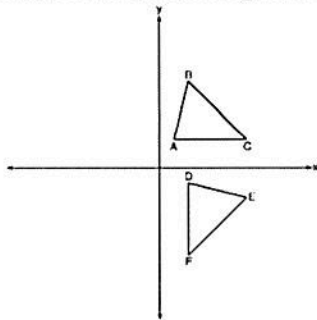


9. The diagram below shows a rotation of θ degrees was performed on $\triangle ABC$ to create $A'B'C'$. If $m\angle A = 52^\circ$ and $m\angle C' = 40^\circ$ find the measure of $\angle B'$. **Explain** your solution.



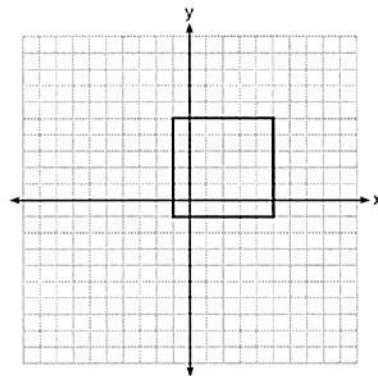
10. The image of $\triangle ABC$ after a rotation of 90° clockwise about the origin is $\triangle DEF$. Which statement is *not* true?

- 1) $\overline{AB} \cong \overline{DE}$
- 2) $\overline{AC} \cong \overline{DF}$
- 3) $\angle C \cong \angle F$
- 4) $\overline{BC} \cong \overline{DE}$

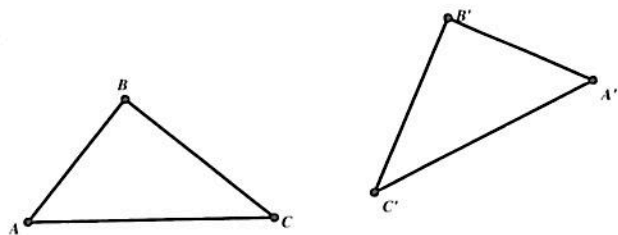


11. In the diagram below, a square is graphed in the coordinate plane. A reflection over which line does *not* carry the square onto itself?

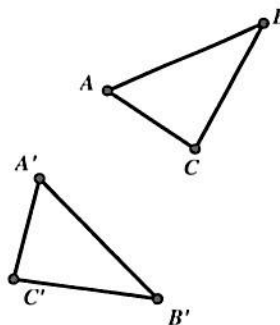
- 1) $y = x$
- 2) $y = 2$
- 3) $x = 5$
- 4) $x = 2$



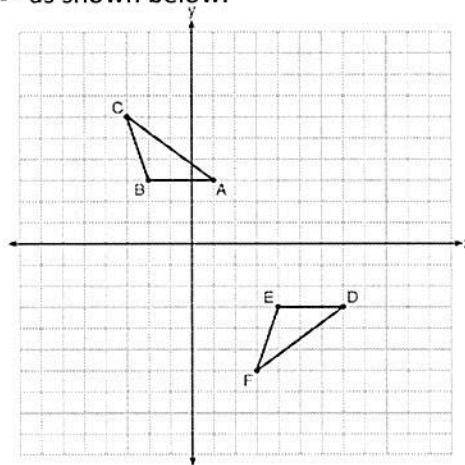
12. Find the line of reflection



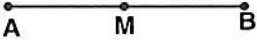
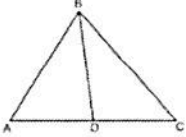
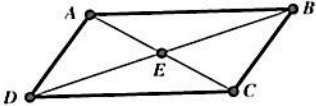
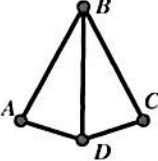
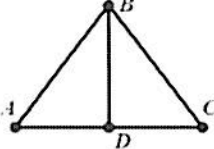
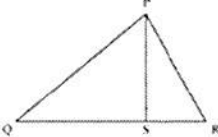
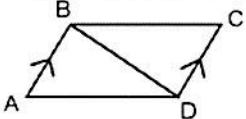
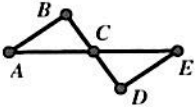
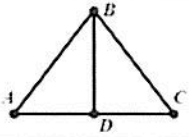
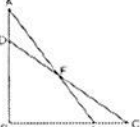
13. Find the center of rotation



14. Describe a sequence of transformations that will map $\triangle ABC$ onto $\triangle DEF$ as shown below.



Topic D – Congruency & Proofs

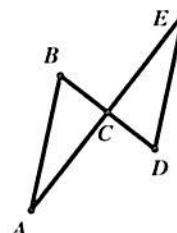
Diagram	Given	Conclusion Statement	Reason
	M is the midpoint of \overline{AB}		A midpoint
	\overline{BD} is a median		A median
	\overline{AC} and \overline{BD} bisect each other at E		A bisector
	\overline{BD} bisects $\angle ABC$		A bisector
	$\overline{BD} \perp \overline{AC}$		Perpendicular lines
	\overline{PS} is an altitude		An altitude
	$\overline{AB} \parallel \overline{CD}$		Given parallel lines cut by a transversal
	The diagram		
	The Diagram		
	The Diagram		

Methods to Prove Triangles are Congruent

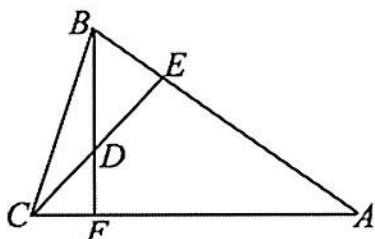
If two triangles are proven congruent we can then say
 "Corresponding _____ or _____ of congruent triangles are _____."

Example 1: Three of the four items listed can be used to establish congruence by ASA. Determine which one is NOT needed to prove $\triangle BCA \cong \triangle DCE$ by ASA?

- (1) C is the midpoint of \overline{AE}
- (2) $\angle B \cong \angle D$
- (3) $\overline{AB} \cong \overline{ED}$
- (4) $\angle A \cong \angle E$



Example 2: Given $\triangle ACE$ and $\triangle ABF$ shown in the diagram to the right, with $\overline{AB} \cong \overline{AC}$. Which statement is needed to prove $\triangle ACE \cong \triangle ABF$ by SAS?

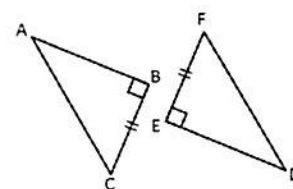


- 1) $\angle ACE \cong \angle ABF$
- 2) $\angle AEC \cong \angle AFB$
- 3) $\overline{BF} \cong \overline{CE}$
- 4) $\overline{AF} \cong \overline{AE}$

HINT: Separate $\triangle ACE$ and $\triangle ABF$

Example 3: To prove $\triangle ABC \cong \triangle DEF$ using Hypotenuse leg, what other information would you need?

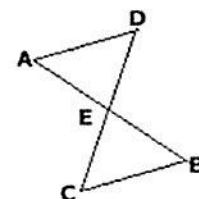
- (1) $\overline{AB} \cong \overline{DE}$
- (2) $\overline{AB} \cong \overline{FD}$
- (3) $\overline{AC} \cong \overline{FD}$
- (4) $\angle ACB \cong \angle DFE$



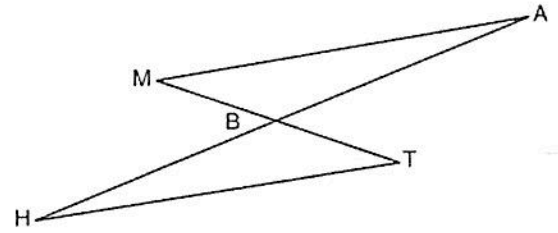
Example 4: Determine congruence from the given information. (1) Create the congruence statement, and then (2) provide the congruence criteria (SSS, SAS, ASA, AAS, HL)

E is the midpoint of \overline{AB} and \overline{DC}

- (1) $\triangle EDA \cong \triangle$ _____
- (2) by _____

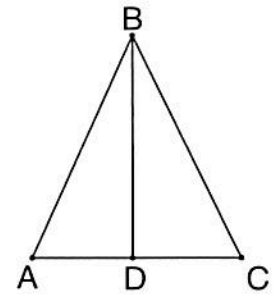


Example 5: Given: $\overline{MA} \parallel \overline{HT}$, and B is the midpoint of \overline{HA} .
Prove: $\triangle MBA \cong \triangle TBH$



Precisely describe a rigid motion that would map $\triangle MBA$ onto $\triangle TBH$.

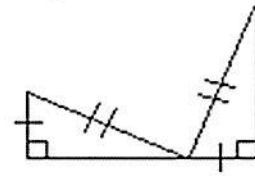
Example 7: Given: $\overline{BD} \perp \overline{AC}$ and \overline{BD} bisects $\angle ABC$
Prove: $\overline{AD} \cong \overline{DC}$



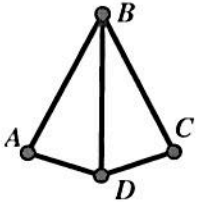
Precisely describe a rigid motion that would map $\triangle ABD$ onto $\triangle CBD$.

Mixed Practice with Topic D Pages 23-24

1. Tiffany notices that two congruent corresponding sides and the corresponding angle and says that these two triangles are congruent by SAS. Is she correct? Are the triangles congruent by SAS or some other congruence criteria? **Explain** your answer.



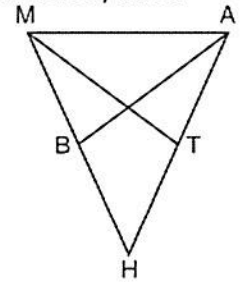
2. Given that \overline{BD} bisects $\angle ADC$, fill in the conclusion statement and reason columns below based on the given.



Conclusion Statement	Reason

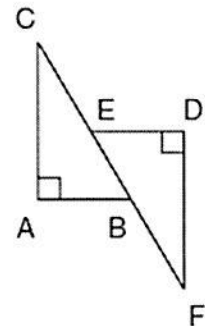
3. In the following diagram Joey is given that $\angle MBA \cong \angle ATM$ and $\angle TMA \cong \angle BAM$. Which sides does Joey know congruent in order for him to complete the proof of $\triangle MBA \cong \triangle ATM$ by AAS congruence criteria.

- (1) $\overline{MH} \cong \overline{AH}$ (3) $\overline{MT} \cong \overline{AB}$
 (2) $\angle BMA \cong \angle MAT$ (4) $\overline{MA} \cong \overline{MA}$



4. In the accompanying diagram, $\overline{CA} \perp \overline{AB}$, $\overline{ED} \perp \overline{DF}$, $\overline{ED} \parallel \overline{AB}$, $\overline{CE} \cong \overline{BF}$, $\overline{AB} \cong \overline{ED}$, and $m\angle CAB = m\angle FDE = 90$. Which criteria could **not** be used to prove $\triangle ABC \cong \triangle DEF$?

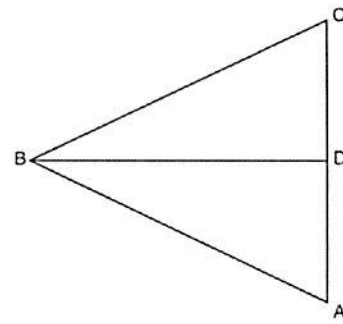
- (1) SSS \cong SSS
 (2) SAS \cong SAS
 (3) AAS \cong AAS
 (4) HL \cong HL



5. In $\triangle BAT$ and $\triangle CRE$, $\angle A \cong \angle R$ and $\overline{BA} \cong \overline{CR}$. Write *one* additional statement that could be used to prove that the two triangles are congruent. State the method that would be used to prove that the triangles are congruent

6. Given: \overline{BD} bisects $\angle ABC$, $\overline{BD} \perp \overline{AC}$

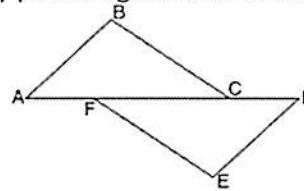
Prove: $\overline{AB} \cong \overline{CB}$



7. Complete the partial proof below for the accompanying diagram by providing reasons for steps 3, 6, 8, 9, 10 and 11.

Given: \overline{AFCD} , $\overline{AB} \perp \overline{BC}$, $\overline{DE} \perp \overline{EF}$, $\overline{BC} \parallel \overline{FE}$, $\overline{AF} \cong \overline{CD}$

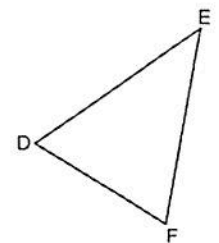
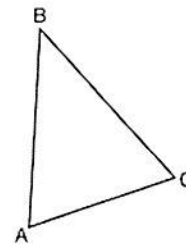
Prove: $\overline{AB} \cong \overline{DE}$



Statements	Reasons
1 \overline{AFCD}	1 Given
2 $\overline{AB} \perp \overline{BC}$, $\overline{DE} \perp \overline{EF}$	2 Given
3 $\angle B$ and $\angle E$ are right angles.	3
4 $\angle B \cong \angle E$	4 All right angles are congruent.
5 $\overline{BC} \parallel \overline{FE}$	5 Given
6 $\angle BCA \cong \angle EFD$	6
7 $\overline{AF} \cong \overline{CD}$	7 Given
8 $\overline{FC} \cong \overline{FC}$	8
9 $\overline{AC} \cong \overline{FD}$	9
10 $\triangle ABC \cong \triangle DEF$	10
11 $\overline{AB} \cong \overline{DE}$	11

8. Which statement is sufficient evidence that $\triangle DEF$ is congruent to $\triangle ABC$?

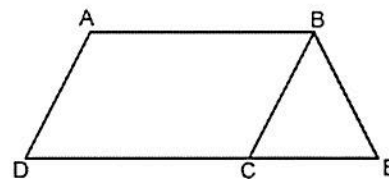
- 1) $AB = DE$ and $BC = EF$
- 2) $\angle D \cong \angle A$, $\angle B \cong \angle E$, $\angle C \cong \angle F$
- 3) There is a sequence of rigid motions that maps \overline{AB} onto \overline{DE} , \overline{BC} onto \overline{EF} , and \overline{AC} onto \overline{DF} .
- 4) There is a sequence of rigid motions that maps point A onto point D, \overline{AB} onto \overline{DE} , and $\angle B$ onto $\angle E$.



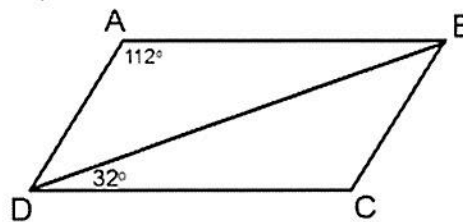
Topic E – Quadrilaterals & Their Properties

	Parallelogram	Rectangle	Rhombus	Square	Trapezoid
At least one pair of opposite sides parallel					
Opposite angles \cong					
Consecutive angles supplementary					
Opposite sides \cong					
Opposite sides parallel					
Diagonals bisect each other					
Diagonals bisect angles					
Diagonals \perp to each other					
Diagonals \cong					
Equiangular					
Equilateral					

Example 1: In the diagram of parallelogram $ABCD$ shown below, \overline{DC} is extended to E , and \overline{BE} is drawn such that $\overline{BC} \cong \overline{CE}$. If $m\angle A = 112^\circ$ what is $m\angle CBE$.



Example 2: The diagram below shows parallelogram $ABCD$ with diagonal \overline{BD} , $m\angle A = 112^\circ$ and $m\angle BDC = 32^\circ$. What is the measure of $\angle ADB$?



- (1) 32°
- (2) 36°
- (3) 144°
- (4) 112°

Example 3: Quadrilateral $MATH$ has diagonals \overline{MT} and \overline{AH} . Which information is *not* sufficient to prove $MATH$ is a parallelogram?

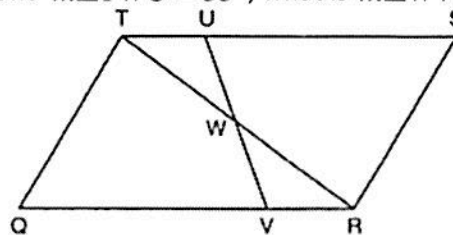
- 1) $\overline{MA} \cong \overline{TH}$ and $\overline{AT} \cong \overline{MH}$
- 2) $\overline{MA} \cong \overline{TH}$ and $\overline{MA} \parallel \overline{TH}$
- 3) \overline{MT} and \overline{AH} bisect each other.
- 4) $\overline{MA} \cong \overline{TH}$ and $\overline{AT} \parallel \overline{MH}$

Example 4: Which of the following group of quadrilaterals have congruent diagonals?

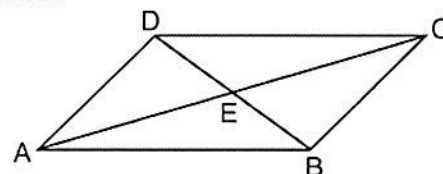
- | | |
|----------------------|-----------------------------------|
| 1) Rhombus, Square | 3) Rhombus, Parallelogram, Square |
| 2) Rectangle, Square | 4) Rectangle, Rhombus, Square |

Example 5: In parallelogram $QRST$ shown below, diagonal \overline{TR} is drawn, U and V are points on \overline{TS} and \overline{QR} , respectively, and \overline{UV} intersects \overline{TR} at W . If $m\angle S = 60^\circ$, $m\angle SRT = 83^\circ$ and $m\angle TWU = 35^\circ$, what is $m\angle WVQ$?

- (1) 37°
- (2) 60°
- (3) 72°
- (4) 83°



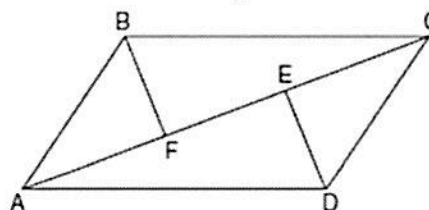
Example 6: Given: In parallelogram $ABCD$ shown below, diagonals \overline{AC} and \overline{BD} intersect at E .
Prove: $\triangle DEA \cong \triangle BEC$



Example 7: Quadrilateral $ABCD$, diagonal \overline{AFEC} is shown in the diagram. Fill in the missing reasons below to complete the following proof.

Given: $\overline{AF} \cong \overline{CE}$, $\overline{BF} \perp \overline{AC}$, $\overline{DE} \perp \overline{AC}$, $\angle BAF \cong \angle DCE$

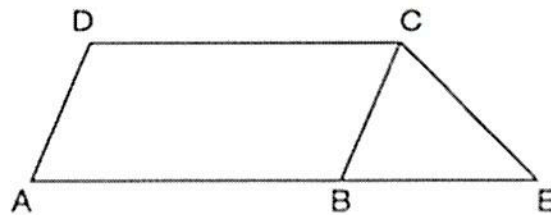
Prove: $ABCD$ is a parallelogram.



STATEMENT	REASON
1. $\overline{AF} \cong \overline{CE}$	1. Given
2. $\overline{BF} \perp \overline{AC}$, $\overline{DE} \perp \overline{AC}$	2. Given
3. $\angle BFA$ and $\angle DEC$ are right angles	3.
4. $\angle BFA \cong \angle DEC$	4. All right angles are congruent
5. $\angle BAF \cong \angle DCE$	5. Given
6. $\overline{BA} \parallel \overline{DC}$	6.
7. $\triangle BAF \cong \triangle DCE$	7.
8. $\overline{BA} \cong \overline{DC}$	8. Corresponding sides of Congruent Triangles are Congruent
9. $ABCD$ is a parallelogram.	9.

Mixed Practice with Topic E Pages 27-28

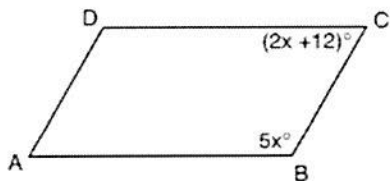
1. In the diagram below, $ABCD$ is a parallelogram, \overline{AB} is extended through B to E and \overline{CE} is drawn. If $\overline{CE} \cong \overline{BE}$ and $m\angle D = 112^\circ$, what is $m\angle E$? Explain your solution.



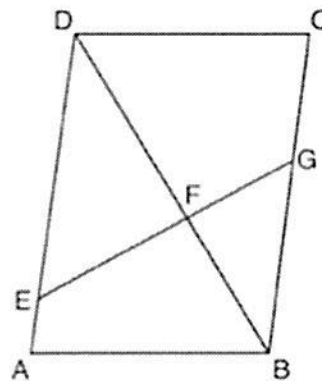
2. Given parallelogram $ABCD$ with diagonals \overline{AC} and \overline{BD} intersecting at E . Which statement must be true?

- 1) $\overline{BE} \cong \overline{CE}$
- 2) $\angle BAE \cong \angle DCE$
- 3) $\overline{AB} \cong \overline{BC}$
- 4) $\angle DAE \cong \angle CBE$

3. In the accompanying diagram of parallelogram $ABCD$, $m\angle B = 5x$, and $m\angle C = 2x + 12$. Find the number of degrees in $\angle D$.

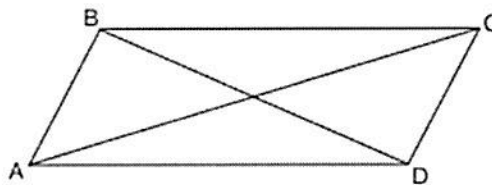


4. Parallelogram $ABCD$, with $m\angle C = 85^\circ$, $m\angle CDF = 52^\circ$ and $m\angle GFB = 80^\circ$ find $m\angle FEA$.



5. Quadrilateral $ABCD$ with diagonals \overline{AC} and \overline{BD} is shown in the diagram below. Which information is *not* enough to prove $ABCD$ is a parallelogram?

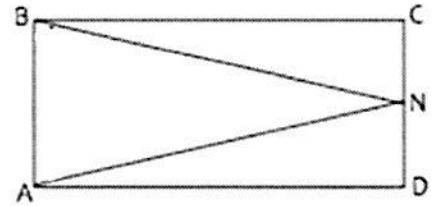
- (1) $\overline{AB} \cong \overline{CD}$ and $\overline{AB} \parallel \overline{CD}$
- (2) $\overline{AB} \cong \overline{CD}$ and $\overline{BC} \cong \overline{AD}$
- (3) $\overline{AB} \cong \overline{CD}$ and $\overline{BC} \parallel \overline{AD}$
- (4) $\overline{AB} \parallel \overline{CD}$ and $\overline{BC} \parallel \overline{AD}$



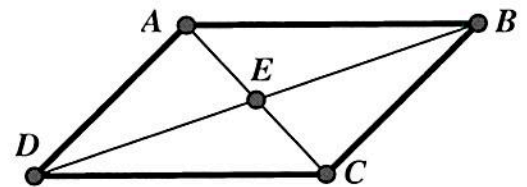
6. In parallelogram $ABCD$, diagonals \overline{AC} and \overline{BD} intersect at E . Which statement does *not* prove parallelogram $ABCD$ is a rhombus?

- 1) $\overline{AC} \cong \overline{DB}$
- 2) $\overline{AB} \cong \overline{BC}$
- 3) $\overline{AC} \perp \overline{DB}$
- 4) \overline{AC} bisects $\angle DCB$

7. Given: Rectangle $ABCD$, with N the midpoint of \overline{CD}
Prove: $\overline{BN} \cong \overline{AN}$



8. Given: In parallelogram $ABCD$ shown below, diagonals \overline{AC} and \overline{BD} intersect at E .
Prove: $\triangle AEB \cong \triangle CED$



1. Which transformation would result in the perimeter of a triangle being different from the perimeter of its image?

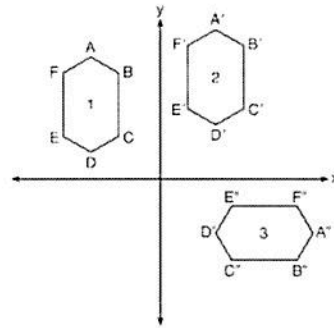
- 1) Translation 2) Dilation 3) Reflection 4) Rotation

2. A parallelogram must be a rhombus when its

- 1) diagonals are perpendicular
 2) opposite sides are parallel
 3) diagonals are congruent
 4) opposite sides are congruent

3. In the diagram below, congruent figures 1, 2, and 3 are drawn. Which sequence of transformations maps figure 1 onto figure 2 and then figure 2 onto figure 3?

- 1) a reflection followed by a translation
 2) a rotation followed by a translation
 3) a translation followed by a reflection
 4) a translation followed by a rotation

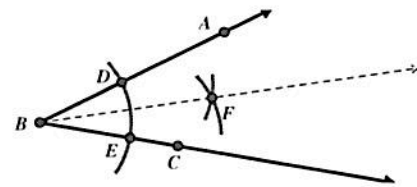


4. Which regular polygon has a minimum rotation of 60° to carry the polygon onto itself?

- 1) Octagon 2) decagon 3) hexagon 4) pentagon

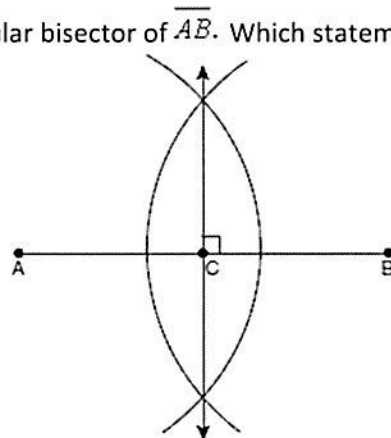
5. Which construction is represented by these construction marks?

- 1) The angle bisector of $\angle ABC$ 3) A perpendicular line \overline{AC}
 2) The perpendicular bisector of \overline{BC} 4) Copying $\angle ABC$



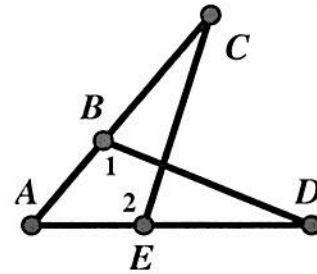
6. The diagram below shows the construction of the perpendicular bisector of \overline{AB} . Which statement is *not* true?

- 1) $AC = CB$
 2) $CB = \frac{1}{2} AB$
 3) $AC = 2AB$
 4) $AC + CB = AB$



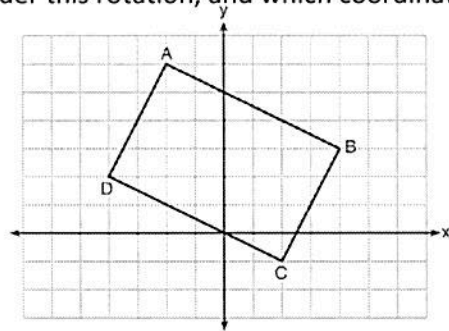
7. As shown in the diagram below $\overline{AB} \cong \overline{AE}$ and $\overline{AC} \cong \overline{AD}$. Which piece of information could be used to prove $\triangle ABD \cong \triangle AEC$ by SAS?

- 1) $\angle ABD \cong \angle AEC$
- 2) $\angle 1 \cong \angle 2$
- 3) $\angle A \cong \angle A$
- 4) $\angle ADB \cong \angle ACE$



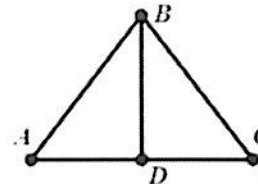
8. Quadrilateral $ABCD$ is graphed on the set of axes below. When $ABCD$ is rotated 90° in a counterclockwise direction about the origin, its image is quadrilateral $A'B'C'D'$. Is distance preserved under this rotation, and which coordinates are correct for the given vertex?

- 1) no and $C'(1, 2)$
- 2) no and $D'(2, 4)$
- 3) yes and $A'(6, 2)$
- 4) yes and $B'(-3, 4)$



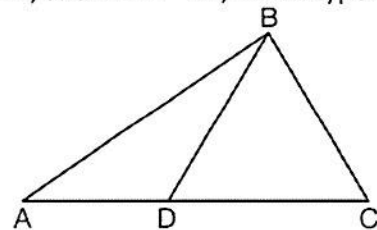
9. Line segment BD is the perpendicular bisector of \overline{AC} , and $\overline{AB} \cong \overline{CB}$. Which conclusion can *not* be proven?

- 1) $\angle A \cong \angle C$
- 2) \overline{BD} is an altitude of triangle ABC .
- 3) Triangle ABC is scalene.
- 4) $\angle BDA = 90^\circ$.

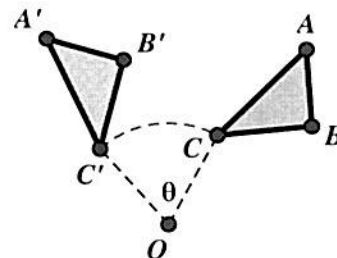


10. In the diagram of $\triangle ABC$, \overline{BD} is drawn to side \overline{AC} . If $m\angle A = 35$, $m\angle ABD = 25$, and $m\angle C = 60$, which type of triangle is $\triangle BCD$?

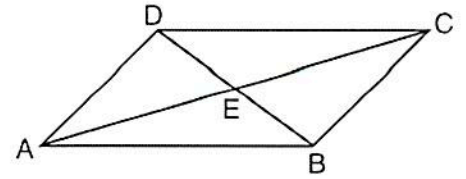
- 1) equilateral
- 2) scalene
- 3) obtuse
- 4) right



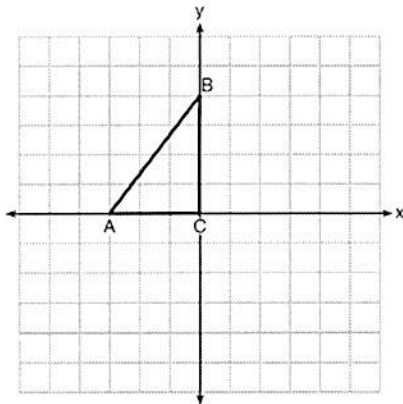
11. The diagram below shows a rotation of θ degrees was performed on $\triangle ABC$ to create $A'B'C'$. If $m\angle A = 52^\circ$ and $m\angle C' = 40^\circ$ find the measure of $\angle B'$. **Explain** your solution.



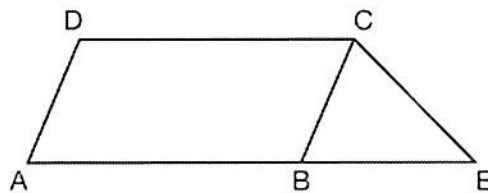
12. Given: In parallelogram $ABCD$ shown below, diagonals \overline{AC} and \overline{BD} intersect at E .
Prove: $\triangle DEA \cong \triangle BEC$



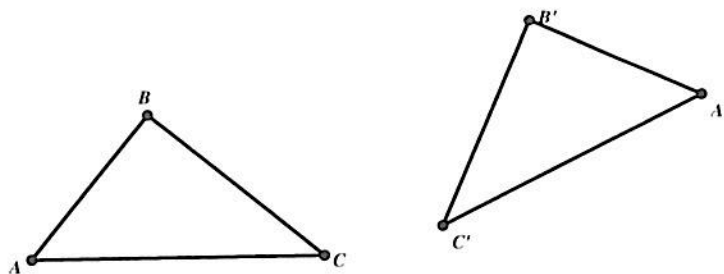
13. Triangle ABC is graphed on the set of axes below. Graph, label and state the coordinates of $\triangle A''B''C''$, the image of $\triangle ABC$ after a reflection over the line $x = 1$ and a translation right 1 down 3.



14. In the diagram below, $ABCD$ is a parallelogram, \overline{AB} is extended through B to E , and \overline{CE} is drawn. If $\overline{CE} \cong \overline{BE}$ and $m\angle D = 112^\circ$, what is $m\angle E$?



15. Construct the line of reflection ℓ for the transformation shown.

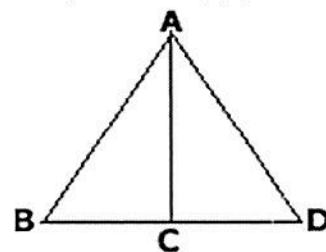


16. Determine congruence from the given information. (1) Create the congruence statement, and then (2) provide the congruence criteria (SSS, SAS, ASA, AAS, HL)

\overline{AC} bisects $\angle BAD$ and $\overline{AC} \perp \overline{BD}$

(1) $\triangle ACB \cong \triangle$ _____

(2) by _____



17. Quadrilateral $ABCD$ is a parallelogram with diagonals \overline{AC} and \overline{BD} intersecting at E . Describe a single rigid motion that maps $\triangle AED$ onto $\triangle CEB$.

