This packet belongs to: Key

CC GEOMETRY Midterm Review January 2020

WHEN/WHERE: Tuesday, January 21. Time: 12 pm in room 120

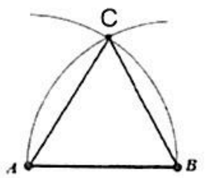
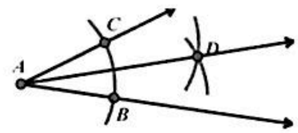
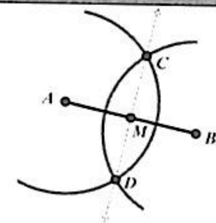
BRING WITH YOU:
- Your graphing calculator
- Compass!
- Two pencils and two pens (black & blue only)
- As much mathematical knowledge as possible

REVIEW:
- Wednesday, 1/15 in class
- Thursday, 1/16 in class
- Friday, 1/17 in class

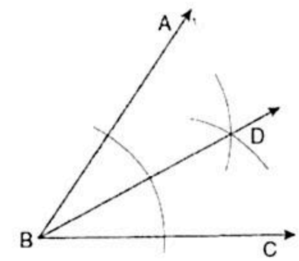
WHAT IT COUNTS FOR:
- Does *not* count as part of your 2nd marking period grade
- Counts as 4% of your final average for the year.

FORMAT:
- 10 Multiple-choice questions (2 points each)
- 5 Short-response, show all work (2 points each)
- 5 Short-response, show all work (4 points each)
- 2 Long-response, show all work questions (6 points each)

TOPIC	THINGS TO STUDY	PAGE(S)
UNIT 1A: CONSTRUCTIONS	• Construct an Equilateral Triangle • Copy and Bisect an Angle • Construct a Perpendicular Bisector • Points of Concurrencies	2
UNIT 1B: UNKNOWN ANGLES	• Solving for Unknown angles (vertical angles, linear pairs, angles at a point, etc.) • Angles in a triangle • Isosceles triangles • Parallel lines and transversals (alternate interior angles, corresponding angles, alternate exterior angles, same side interior angles) • Exterior angle theorem	2-3
UNIT 2: TRANSFORMATIONS/RIGID MOTIONS	• Rotations, reflections, translations • Symmetry- Reflectional, Rotational (Angles of Rotation) • Sequence of rigid motions • Transformations on the coordinate plane • Construct line of reflection • Congruence in terms of rigid motions	3--6
UNIT 3: TRIANGLE CONGRUENCE	• Congruence Criteria—SAS, ASA, SSS, SAA and HL, • CPCTC	6-7
UNIT 4: QUADRILATERALS	• Properties of quadrilaterals • Parallelogram proofs	8-10
UNIT 5: SIMILARITY	• Scale drawings (Constructing Dilations) • Scale factors • Similarity Transformations • Similarity Theorems (AA, SAS, SSS) • Side Splitter • Dilating a line • Similarity proofs	11-15

Identify the following Basic Constructions		
		
equilateral Δ	angle Bisector	⊥ bisector

1. Based on the construction below, which statement must be true?



∡ bisector !

~~1)~~ $m\angle ABD = \frac{1}{2} m\angle CBD$

2) $m\angle ABD = m\angle CBD$

~~3)~~ $\frac{1}{2} m\angle ABD = m\angle ABC$

~~4)~~ $m\angle CBD = \frac{1}{2} m\angle ABD$

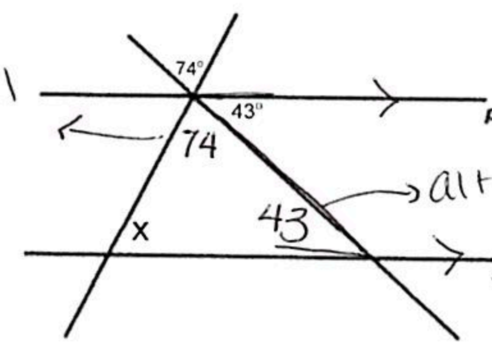
Vertical Angles	Angle Sum of a Triangle	Angles on a Line	Parallel Lines cut by Transversal	Isosceles Triangle
Vertical Angles are ≅.	Angles in a triangle add to 180 °.	Angles on a line add to 180 °.	Alternate Interior angles are ≅. Z Corresponding angles are ≅. F	2 = sides. 2 = base angles.

2. In the diagram below, $p \parallel s$. Determine the value of x.

$180-(74+43) = \boxed{63°}$

B/C ∡'s in a Δ sum to 180

vertical ∡'s



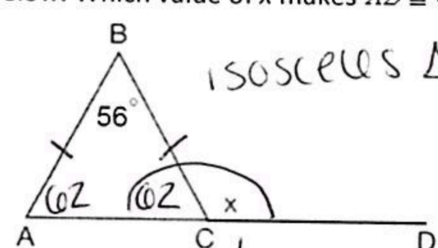
alt. int. ∡'s

3. Given $\triangle ABC$ with $m\angle B = 56°$ and side $\overline{AC}$ extended to D, as shown below. Which value of x makes $\overline{AB} \cong \overline{CB}$?

1) 59º
2) 62º
3) 118º
4) 121º

$180-56 = 124°$

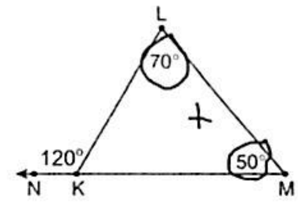
$124 \div 2 = 62°$



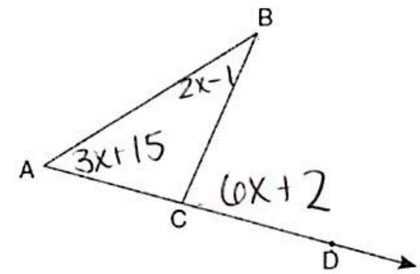
isosceles Δ !

$180-62 = 118°$

linear pair

Exterior Angle Theorem
The measure of an ***exterior angle*** of a triangle is equal to the sum of the measures of the two non-adjacent interior angles of the triangle.  $m\angle LKN = m\angle L + m\angle M$

4. In the diagram below, ΔABC is shown with $\overline{AC}$ extended through point D. If $m\angle BCD = 6x + 2$, $m\angle BAC = 3x + 15$, and $m\angle ABC = 2x - 1$, what is the measure of $\angle ABC$? Explain your solution.



$6x+2 = 3x+15+2x-1$

$6x+2 = 5x+14$

$-5x-2 \quad -5x \quad -2$

$x = 12$

$\angle ABC = 2(12)-1$

$= 24-1$

$= \boxed{23°}$

The exterior ∡ of a Δ ='s the sum of the 2 non-adj. int. ∡'s

Rigid Motions			
Preserve distance and ∡ measure.	Reflection	Rotation	Translation

5. The image of $\triangle DEF$ is $\triangle D'E'F'$. Under which transformation will the triangles *not* be congruent?

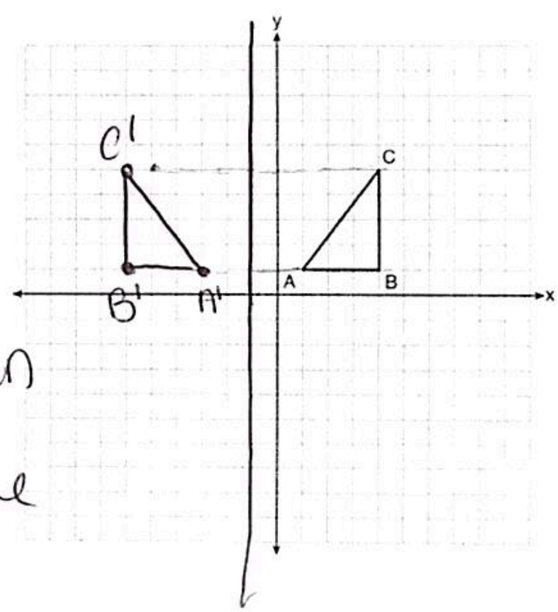
1) a reflection through the origin
2) a reflection over the line $y = x$
3) a dilation with a scale factor of 1 centered at $(2, 3)$
4) a dilation with a scale factor of $\frac{3}{2}$ centered at the origin

Horizontal Lines	Vertical Lines
Equation in the form y = #	Equation in the form x = #

6. Triangle *ABC* is graphed on the set of axes below. Graph and label $\triangle A'B'C'$, the image of $\triangle ABC$ after a reflection over $x = -1$.

vertical line!

all corresponding points are equidistant from line of reflection



Explain why $\triangle ABC \cong \triangle A'B'C'$:

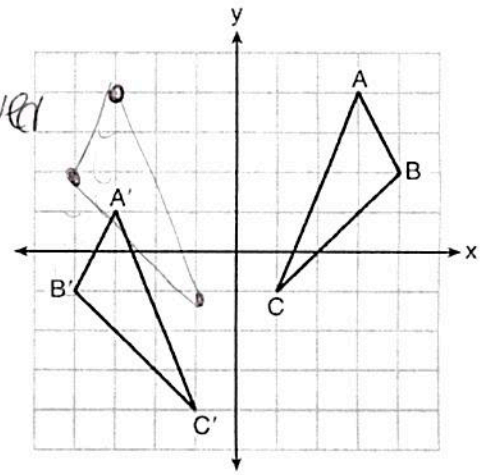
ΔABC ≅ ΔA'B'C' b/c a reflection is a rigid motion which preserves distance + ∡ measure

Describing Rigid Motions!		
Reflection needs • Line or point of reflection	Rotation needs • Center • Angle (#degrees) • Direction (counter-clockwise is positive, clockwise is negative)	Translation needs • A vector with distance and direction

7. As graphed on the set of axes below, $\triangle A'B'C'$ is the image of $\triangle ABC$ after a sequence of transformations.

a) Determine and state the sequence of transformations.

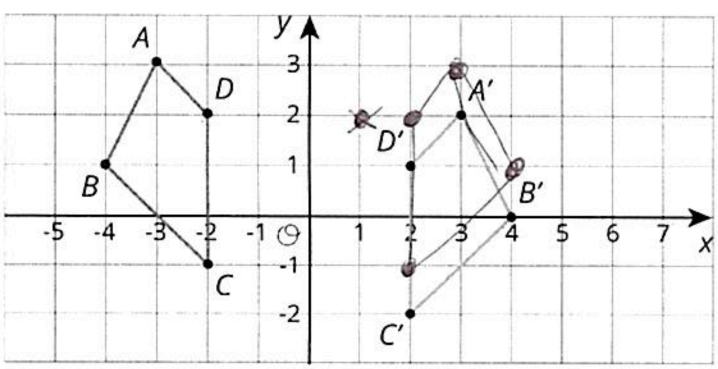
a reflection over the y-axis followed by a translation of 3 units down



b) Is $\triangle A'B'C'$ congruent to $\triangle ABC$? Use the properties of rigid motions to explain your answer.

yes b/c a reflection & translation are both rigid motions which preserve distance and ∠ measure

c) Determine and state the sequence of transformations that mapped trapezoid ABCD to A'B'C'D'.

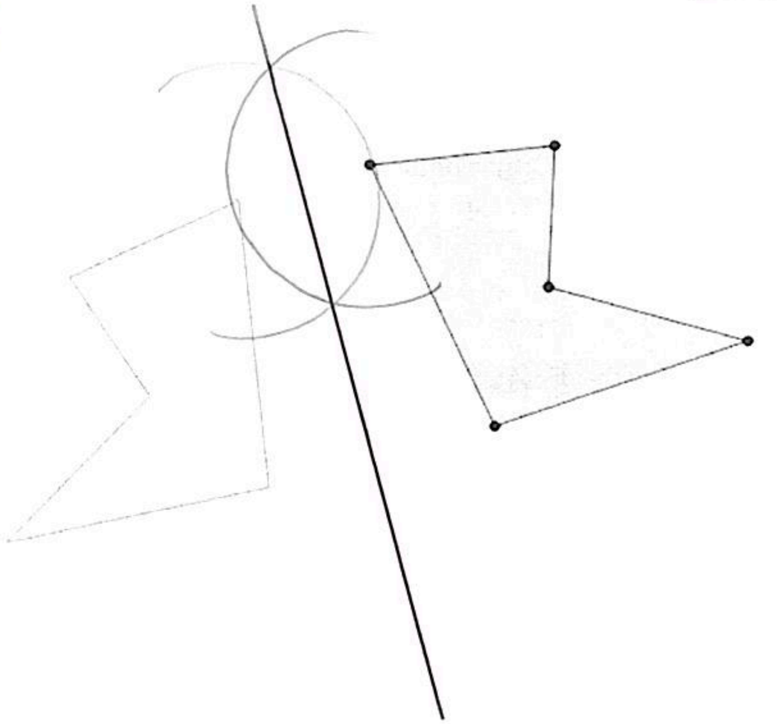


a reflection over the y-axis followed by a translation of down 1 unit.

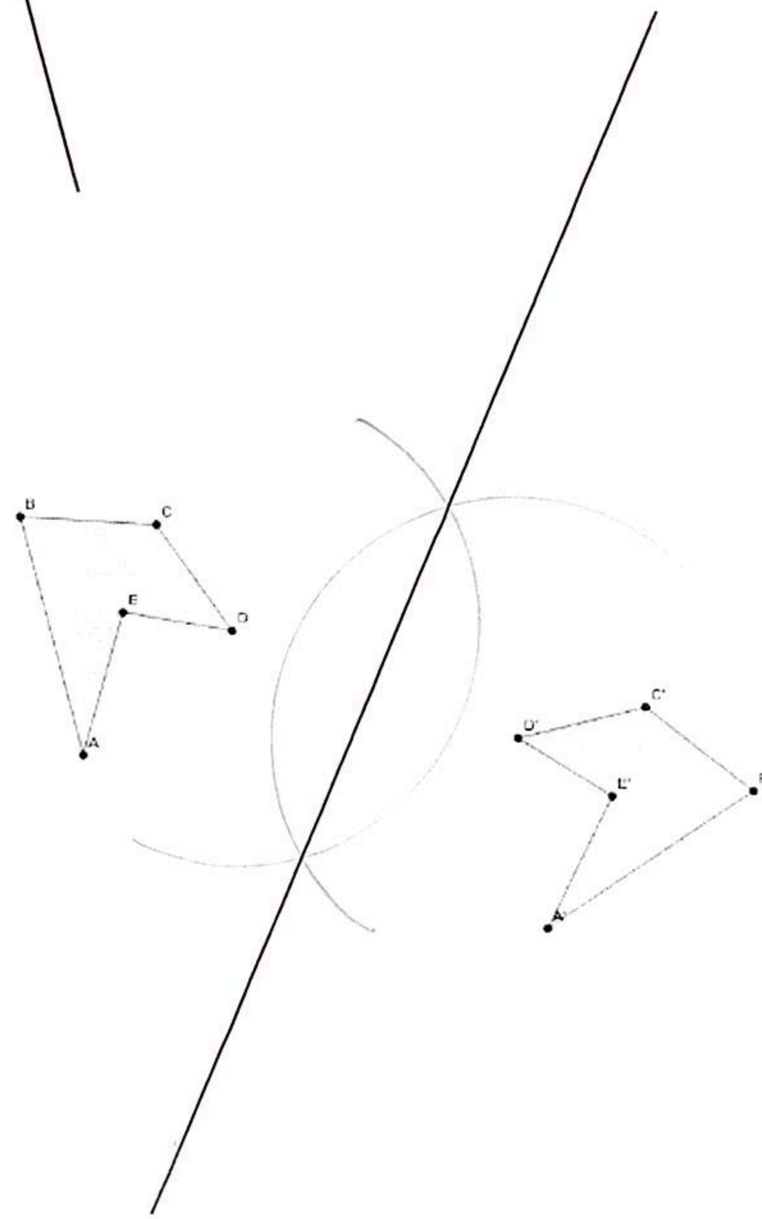
Using a compass and a straight edge, construct a line of reflection for the following figures:

↳ ⊥ bisector!

A.



B.



Formula to determine the MINIMUM Rotation for a REGULAR polygon to map onto itself
minimum $\angle = \dfrac{360}{\# sides}$ Any <u>multiple</u> of this angle will also map the polygon onto itself.

8. A regular hexagon is rotated *n* degrees about its center, carrying the hexagon onto itself. The value of *n* could be

1) 30°
2) 60°
3) 140°
4) 150°

$\frac{360}{6} = 60°$

9. A regular decagon is rotated *n* degrees about its center, carrying the decagon onto itself. The value of *n* could be

~~1)~~ 10°
~~2)~~ 150°
~~3)~~ 225°
4) 252°

} not multiples

$\frac{360}{10} = 36$

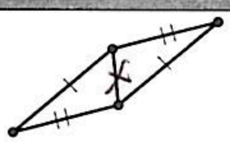
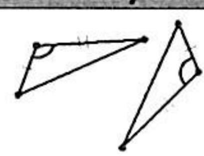
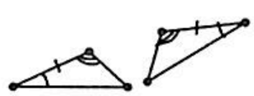
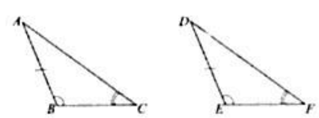
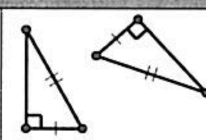
$\frac{252}{36} = 7$ so 252 is a multiple of 36!

10. Which polygon has a minimum rotation of 72° about its center to carry the polygon onto itself?

1) square $\frac{360}{4} = 90$
2) pentagon $\frac{360}{5} = 72$
3) heptagon $\frac{360}{6} = 60$
4) octagon $\frac{360}{8} = 45$

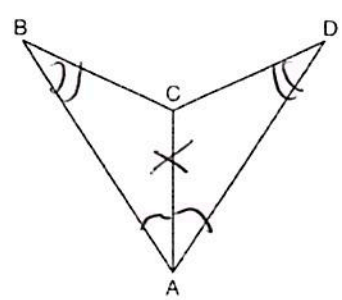
$\frac{360}{n} = 72$ Guess & Check!

TRIANGLE PROOFS

Identify the 5 Methods to Prove Triangles are Congruent				
				
SSS	SAS	ASA	AAS	HL

VISUAL FREEBIES → vertical ∠'s and Reflexive

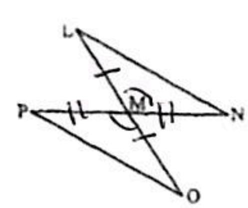
12. As shown in the diagram below, $\overline{AC}$ bisects $\angle BAD$ and $\angle B \cong \angle D$. Which method could be used to prove $\Delta ABC \cong \Delta ADC$?



1) $ASA \cong ASA$
2) $AAS \cong AAS$
3) $SAS \cong SAS$
4) $SSS \cong SSS$

13. In the accompanying diagram, M is the midpoint of $\overline{LO}$ and $\overline{NP}$. Which triangle congruency can be used to prove $\Delta LMN \cong \Delta OMP$?

(1) $AAA \cong AAA$
(2) $SSS \cong SSS$
(3) $ASA \cong ASA$
(4) $SAS \cong SAS$



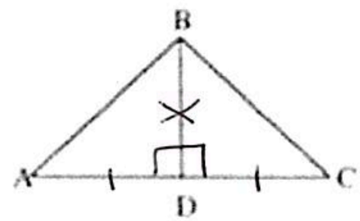
PROOF PRACTICE:

14. Given: $\overline{BD} \perp \overline{AC}$

D is the midpoint of $\overline{AC}$

Prove: $\Delta ABD \cong \Delta CBD$

Plan: SAS

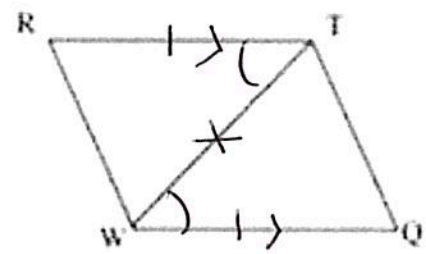


STATEMENT	REASON
① $\overline{BD} \perp \overline{AC}$, D is midpoint of $\overline{AC}$	① Given
② $\overline{AD} \cong \overline{DC}$ Ⓢ ✓	② midpoints create 2 $\cong$ segments
③ $\angle ADB \cong \angle CDB$ Ⓐ ✓	③ $\perp$ lines create $\cong$ right $\angle$'s
④ $\overline{BD} \cong \overline{BD}$ Ⓢ ✓	④ Reflexive property
⑤ $\Delta ABD \cong \Delta CBD$	⑤ SAS $\cong$ SAS

15. Given: $\overline{RT} \cong \overline{WQ}$ and $\overline{RT} \parallel \overline{WQ}$

Prove: $\overline{RW} \cong \overline{TQ}$ → CPCTC!

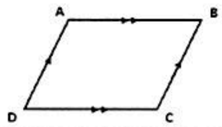
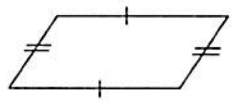
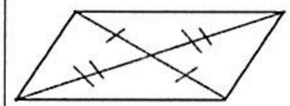
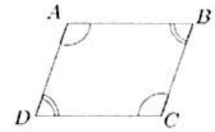
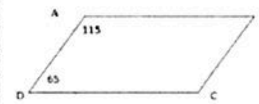
Plan: SAS ✓



STATEMENT	REASON
① $\overline{RT} \cong \overline{WQ}$ Ⓢ ✓ + $\overline{RT} \parallel \overline{WQ}$	① Given
② $\angle RTW \cong \angle QWT$ Ⓐ	② alt. int. $\angle$'s are $\cong$
③ $\overline{WT} \cong \overline{WT}$	③ Reflexive property
④ $\Delta RTW \cong \Delta QWT$	④ SAS $\cong$ SAS
⑤ $\overline{RW} \cong \overline{TQ}$	⑤ CPCTC

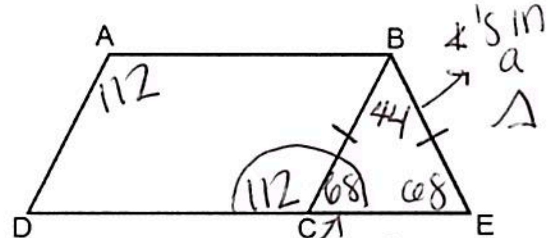
QUADRILATERAL PROOFS

Under each diagram state the property of a parallelogram being illustrated by the labels of the diagram.

Parallelogram Properties				
				
opp. sides are ∥	opp. sides are ≅	Diagonals Bisect each other	opp. ∡'s are ≅	consecutive ∡'s are supp

16. In the diagram of parallelogram $ABCD$ shown below, $\overline{DC}$ is extended to *E*, and $\overline{BE}$ is drawn such that $\overline{BC} \cong \overline{BE}$. If $m\angle A = 112°$. Determine the measure of $\angle EBC$.

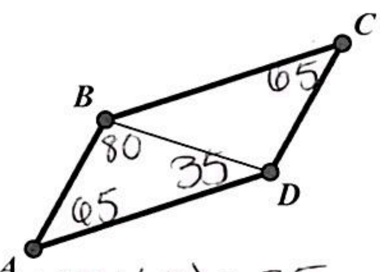
$\boxed{\angle EBC = 44°}$



180 − 112 = 68
linear pair

17. The diagram below shows parallelogram $ABCD$ with diagonal $\overline{BD}$, $m\angle C = 65°$, and $m\angle ABD = 80°$. Determine the following angle measures and explain each:

Angle Measure	Explanation
$m\angle A =$ 65	opp. ∡'s of a ▱ are ≅
$m\angle ADB =$ 35°	∡'s in a Δ sum to 180°

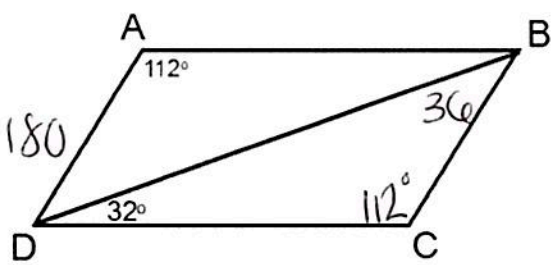


180 − (80 + 65) = 35

18. The diagram below shows parallelogram *ABCD* with diagonal $\overline{BD}$, $m\angle A = 112°$ and $m\angle BDC = 32°$. What is the measure of $\angle CBD$? Explain any property used to reach your solution.

∡C = 112 b/c opp. ∡'s are ≅

∡DBC = 36° b/c ∡'s in a Δ sum to 180



PROOF PRACTICE:

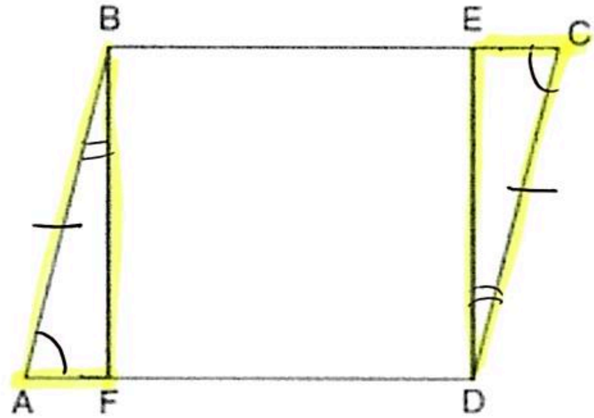
18. Given: Parallelogram *ABCD*, $\measuredangle ABF \cong \measuredangle CDE$

Prove: $AF \cong EC$

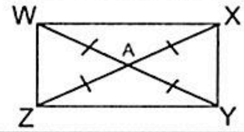
$\triangle ABF \cong \triangle CDE$

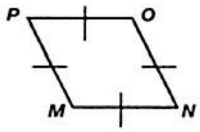
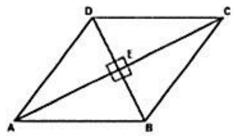
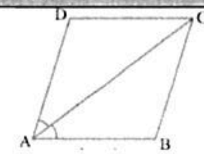
✓✓✓

PLAN: ASA



STATEMENT	REASON
1. ▱ ABCD, $\measuredangle ABF \cong \measuredangle CDE$ (A)	1. Given
2. $\overline{AB} \cong \overline{CD}$ (S)	2. Opp. sides of a ▱ are ≅
3. $\measuredangle A \cong \measuredangle C$ (A)	3. Opp. ∡'s of a ▱ are ≅
4. $\triangle ABF \cong \triangle CDE$	4. ASA ≅ ASA
5. $\overline{AF} \cong \overline{EC}$	5. CPCTC

Rectangle has all the properties of a parallelogram PLUS:	
	
4 right ∡'s	congruent diagonals

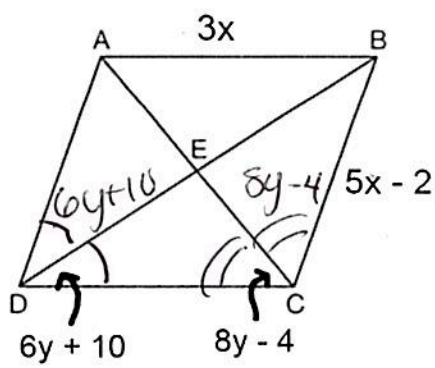
Rhombus has all the properties of a parallelogram PLUS:		
		
4 ≅ sides	⊥ bisector diagonals	Diagonals bisect the angles

17. Which of the following group of quadrilaterals have congruent diagonals?

~~I.~~ Parallelogram
II. Rectangle ✓
~~III.~~ Rhombus
IV. Square ✓

1) All of the above
2) II, III and IV
3) II and IV, only
4) III and IV, only

18. In the diagram below, quadrilateral *ABCD* is a rhombus with diagonals $\overline{AC}$ and $\overline{BD}$ intersecting at *E*. $AB = 3x$, $BC = 5x - 2$, $m\angle CDB = 6y + 10$, and $m\angle DCA = 8y - 4$



(a) Find x: all sides are ≅

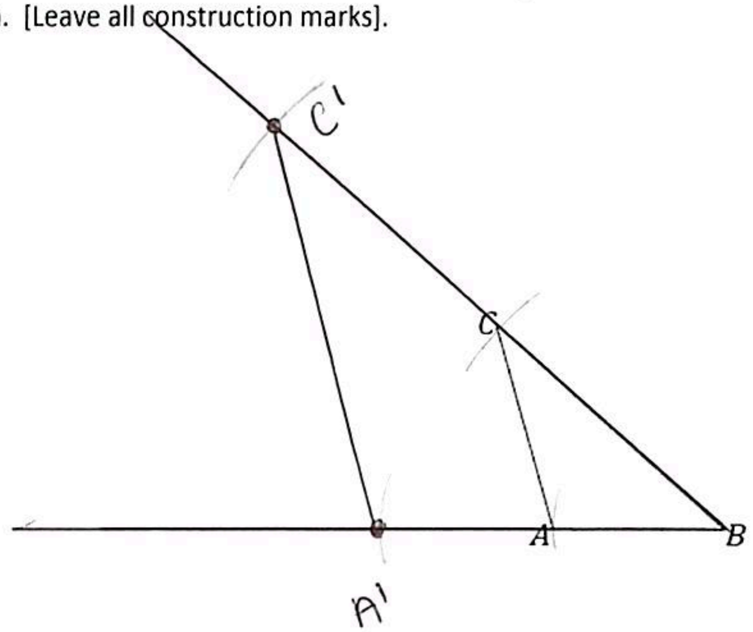
$3x = 5x - 2$
$-5x \quad -5x$
$-2x = -2$
$\boxed{x = 1}$

(b) Find y: Diagonals bisect ∡'s & consecutive ∡'s are supp.

$2(6y+10) + 2(8y-4) = 180$
$12y + 20 + 16y - 8 = 180$
$28y + 12 = 180$
$28y = 168$
$\boxed{y = 6}$

Dilations

19. Use construction tools to create a scale drawing of $\triangle ABC$ with a scale factor of $k = 2$. Use B as the center of dilation. [Leave all construction marks].



Steps to Construct a Dilation
with $k > 1$

1. Use ruler to make a line from center through any vertex(A) and continue past the vertex.

2. Bullseye on center(B), measure to vertex (A) , make a small arc.

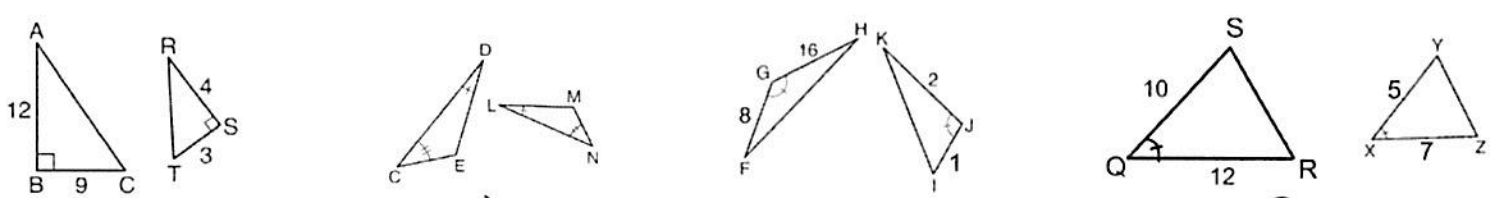
3. Move bullseye to small arc at A, keep frozen and make 2nd small arc on extended online.

4. This will be A' for a dilation of 2. (Repeat small arcs if $k > 2$)

A dilation is a ___Similarity___ transformation. A dilation preserves ___$\measuredangle$___ measure.

3 ways to prove triangles are similar			Similar triangles
$AA \sim$ $\cong$	$SAS \sim$	$SSS \sim$	Corresponding Sides are _in_ _proportion_ Corresponding angles are _congruent_.

20. Using the information given below, which set of triangles, **cannot** be proven similar?



~~(1)~~ ~~(2)~~ ~~(3)~~ (4)

$\frac{12}{4} = \frac{9}{3}$ ✓

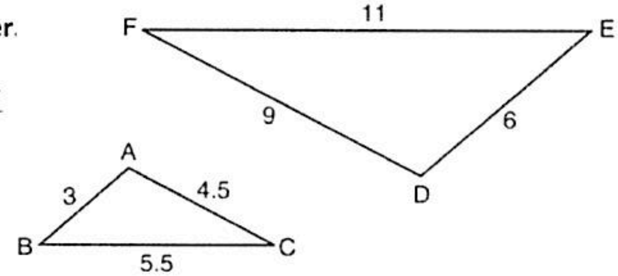
AA ✓

$\frac{8}{1} = \frac{16}{2}$ ✓

$\frac{10}{5} \neq \frac{12}{7}$

sides are not in proportion!

21. Based on the diagram shown, is $\Delta ABC \sim \Delta DEF$? **Justify your answer**.

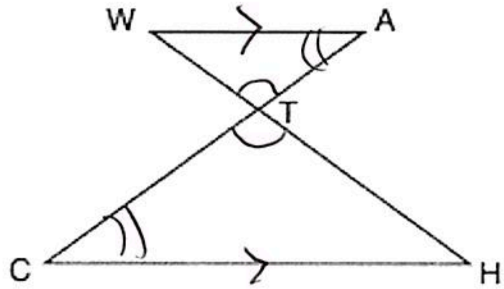


S	$\frac{3}{6} = .5$ ✓
S	$\frac{4.5}{9} = .5$ ✓
S	$\frac{5.5}{11} = .5$ ✓

$\Delta ABC \sim \Delta DEF$
by SSS~

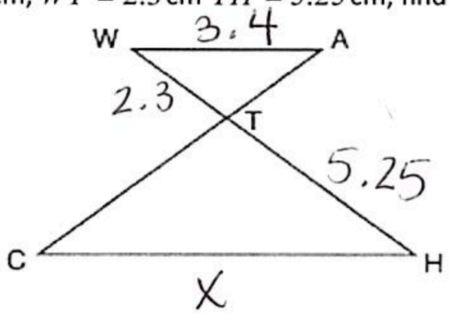
22. In the accompanying diagram, $\overline{WA} \parallel \overline{CH}$ and $\overline{WH}$ and $\overline{AC}$ intersect at point *T*.

a) Prove that $\Delta WAT \sim \Delta HCT$.



Statement	Reason
① $\overline{WA} \parallel \overline{CH}$, $\overline{WH}$ & $\overline{AC}$ intersect @ T	① Given
② $\angle WTA \cong \angle HTC$ (A)	② vertical ∠'s are ≅
③ $\angle A \cong \angle C$ (A)	③ alt. int. ∠'s are ≅
④ $\Delta WAT \sim \Delta HCT$	④ AA ≅ AA

b) Use the information from above and the diagram shown. Given $WA = 3.4$ cm, $WT = 2.3$ cm $TH = 5.25$ cm, find the length of CH to the nearest hundredth of a centimeter.



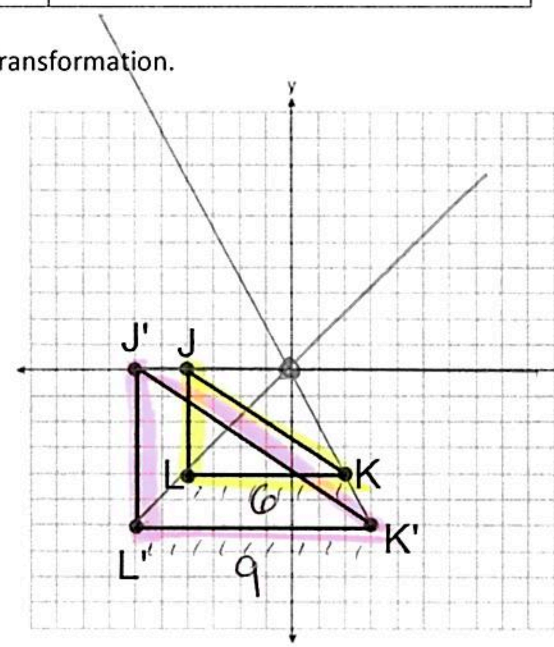
$$\frac{2.3}{5.25} = \frac{3.4}{x}$$

$$2.3x = 17.85$$

$$\boxed{x = 7.76\text{ cm}}$$

To find Center of Dilation	To Find Scale Factor	Describing the dilation
Connect 2 pairs of corresponding points and find the point of intersection.	$k = \frac{\text{new}}{\text{old}}$ (count lengths using only vertical/horizontal segments)	**Need:** **Center** **Scale Factor**

23. In the diagram below, $\triangle A'B'C'$ is the image of $\triangle ABC$ after a SINGLE transformation.



a) Precisely describe the single transformation that was performed.

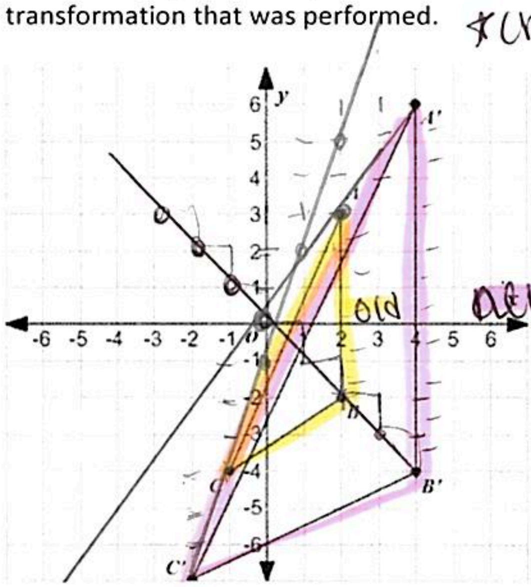
A dilation about (0,0) with a scale factor of $\frac{3}{2}$

b) Explain why ΔJKL is similar to $\Delta J'K'L'$.

$\Delta JKL \sim \Delta J'K'L'$ b/c the sides are in proportion!

$K = \frac{9}{6} = \frac{3}{2}$

24. In the diagram below, $\triangle A'B'C'$ is the image of $\triangle ABC$ after a SINGLE transformation. Precisely describe the single transformation that was performed. *change for next year!*



new

$K = \frac{10}{5} = 2$

A dilation centered @ (0,0) with a scale factor of 2

Dilating Lines and Segments	
If CENTER ON THE LINE **Keep equation the SAME.**	If CENTER OFF THE LINE **Keep the slope the same.** (because image is parallel to pre-image) **Multiply the y - intercept by the scale factor (k).**

25. Line *a* is represented by the equation $5x+2y=14$. Write equation in $y = mx + b$ form.

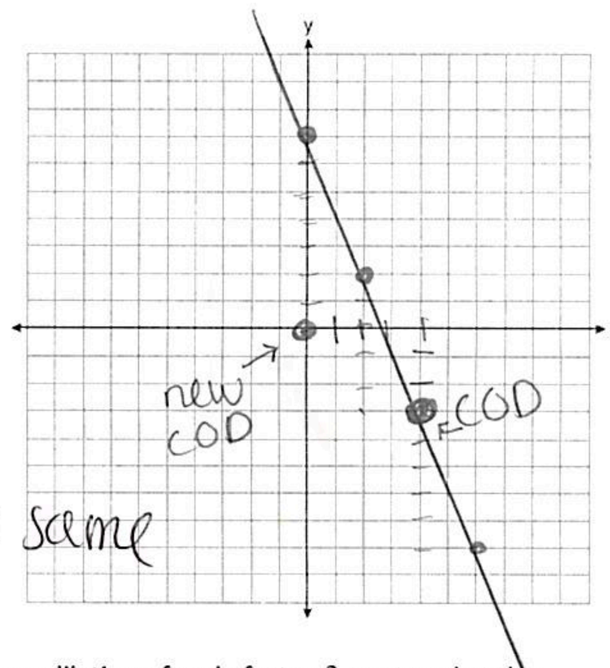
$$-5x \quad -5x$$

$$\frac{2y}{2} = \frac{-5x}{2} + \frac{14}{2}$$

$$\boxed{y = -\frac{5}{2}x + 7}$$

a) Determine and state the equation of line *p*, the image of line *a*, after a dilation of scale factor $\frac{1}{5}$ centered at the point $(4,-3)$. [The use of the set of axes below is optional.]

$$\boxed{y = -\frac{5}{2}x + 7}$$



b) Explain your answer.

If the C.O.D is on the line, the equation remains the same

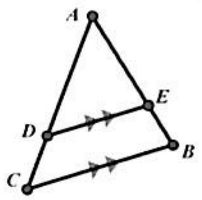
c) Determine and state the equation of line *q*, the image of line *a*, after a dilation of scale factor 3 centered at the origin.

(multiply y-int by K)

$7 \times 3 = 21$

new 'b'

$$\boxed{y = -\frac{5}{2}x + 21}$$

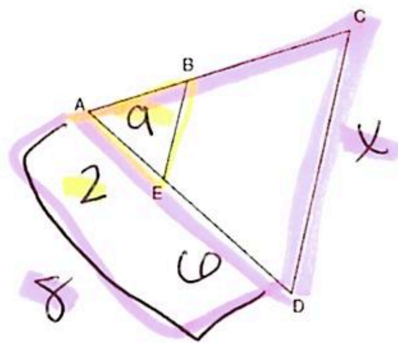
Triangle Side Splitter working with BASES($\parallel$ sides)	**Steps to Solve problems involving the bases (parallel sides)**
	• Separate the Big Δ and the small Δ • Create a proportion using corresponding sides

26. In the diagram below of $\triangle ACD$, *E* is a point on $\overline{AD}$ and *B* is a point on $\overline{AC}$, such that $\overline{EB} \parallel \overline{DC}$. If $AE = 2$, $DE = 6$, and $EB = 9$, find the length of $\overline{CD}$.

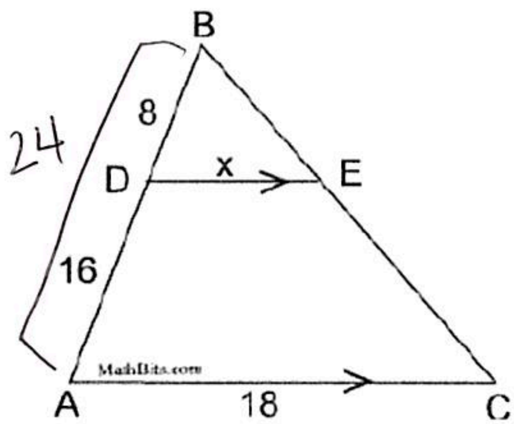
$$\frac{2}{9} = \frac{8}{x}$$

$$2x = 72$$

$$\boxed{x = 36}$$



27. Solve for x:



$$\frac{8}{x} = \frac{24}{18}$$

$$144 = 24x$$

$$\boxed{x = 6}$$